Grounding Thin-Air Reads with Event Structures
Technical Appendix

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This is the technical appendix of the article "Grounding Thin-Air Reads with Event Structures." It contains the proofs of the simulation of the promising semantics by WEAKEST, DRF theorems, and various compilation correctness results along with the evaluation of the proposed models on the Java causality testcases and the construction rules of WEAKESTMO-LLVM.

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A PROVING SIMULATION OF PROMISING SEMANTICS BY WEAKEST

We restate the definition of simulation relation.

Definition 6. Let $\mathbb{P}$ be a program with T threads, $\Pi \subseteq \mathsf{T}$ be a subset of threads, G be a WEAKEST event structure, and $\mathsf{MS} = \langle \mathcal{TS}, \mathcal{S}, \mathsf{M}\rangle$ be a promise machine state. We say that $G \sim_{\Pi} \mathsf{MS}$ holds iff there exist $\mathbb{W}$, $\mathbb{S}$, and sc such that the following conditions hold:

(1) G is consistent according to the WEAKEST model: $\mathsf{isCons}_{\text{WEAKEST}}(G)$.
(2) The local state of each thread in MS contains the program of that thread along with the sequence of covered events of that thread: $\forall i.\ \mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
(3) Whenever $\mathbb{W}$ maps an event of G to a message in MS, then the location accessed and the written values match: $\forall e \in \mathsf{dom}(\mathbb{W}).\ e.\mathsf{loc} = \mathbb{W}(e).\mathsf{loc} \wedge e.\mathsf{wval} = \mathbb{W}(e).\mathsf{wval}$.
(4) All outstanding promises of threads $(\mathsf{T} \setminus \Pi)$ have corresponding write events in G that are po-after $\mathbb{S}$: $\forall i \in (\mathsf{T} \setminus \Pi).\ \forall e \in (\mathbb{S}_0 \cup \mathbb{S}_i).\ \mathcal{TS}(i).\mathsf{P} \subseteq \{\mathbb{W}(e') \mid (e, e') \in G.\mathsf{po}\}$.
(5) For every location x and thread i, the thread view of x in the promise state MS records the timestamp of the maximal write visible to the covered events of thread i.

$$\forall i, x.\ \mathcal{TS}(i).V(x) = \max\{\mathbb{W}(e).\mathsf{ts} \mid e \in \mathsf{dom}([\mathcal{W}_x]; G.\mathsf{jf}^?; \mathsf{shb}^?; \mathsf{sc}^?; \mathsf{shb}^?; [\mathbb{S}_i])\}$$

(6) The $\mathbb{S}$ events satisfy coherence: $\mathsf{shb}; \mathsf{seco}^?$ is irreflexive.
(7) The atomicity condition holds for the $\mathbb{S}$ events: $\mathsf{sfr}; \mathsf{smo}$ is irreflexive.
(8) The sc fences are appropriately ordered by sc: $[\mathcal{F}_{\mathsf{sc}}]; (\mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}); [\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}$.
(9) The behavior of MS matches that of the $\mathbb{S}$ events: $\mathsf{Behavior}(\mathsf{MS}) = \mathsf{Behavior}(G, \mathbb{W}, \mathbb{S})$.

Before proceeding further we introduce certain definition and observations which we use in the proofs.

Auxiliary Definitions.

- We define *immediate* relation: given a relation R we use $\mathsf{imm}(R)$ to denote the immediate edges of R, that is, $\mathsf{imm}(R) \triangleq R \setminus (R\,;R)$.
- Given the Behavior, $\mathsf{Behavior}|_x$ denotes the $\{(x, v)\}$ where v is the value at location x.
- We define swe the external synchronization relation, that is, $\mathsf{swe} \triangleq \mathsf{sw} \setminus \mathsf{po}$.
- In the following discussion op_a denotes the promise machine state transition operation which results in event a in the event structure and the promise machine reaches machine state MS_a.
- $\mathbb{EW}$ denotes the set of read write events where a write is $\mathbb{W}$-mapped to some PS message or a read reads from a $\mathbb{W}$-mapped write.

$$\mathbb{EW} \triangleq \{e \in G.\mathsf{E} \mid e \in \mathcal{W} \cap \mathsf{dom}(\mathbb{W}) \vee \exists w \in \mathsf{dom}(\mathbb{W}).\ G.\mathsf{rf}(w, e)\}$$

- $\mathsf{ts}(e)$ returns the timestamp of a write or view of a read on the respective locations.

$$\mathsf{ts}(e) \triangleq \begin{cases} \mathbb{W}(e).ts & \text{if } e \in \mathsf{St} \cap \mathbb{EW} \\ \mathbb{W}(w).ts & \text{if } e \in \mathsf{Ld} \cap \mathbb{EW} \text{ and } G.\mathsf{rf}(w, e) \end{cases}$$

- In the promise machine cur, rel, acq denotes the current, release, acquire thread views similar to Kang et al. [2017]. The cur view is default.

Additionally, we enlist certain observations regarding the relation between the promise machine and event structure.

Observations. Considering the promising semantics and event structure we observe the followings.

(1) The $(G.\mathsf{E} \setminus \mathbb{S})$ events correspond to the certificate steps of a promise. The certificate steps do not have any release or fence operations. Hence there is no release or fence event in $(G.\mathsf{E} \setminus \mathbb{S})$.

As a result, these events do not have outgoing $G.\mathsf{sw}$ edges. Hence the source event of an incoming $G.\mathsf{sw}$ edge is in $\mathbb{S}$, that is, $G.\mathsf{sw} \subseteq (\mathbb{S} \times G.\mathsf{E})$. Also for $(G.\mathsf{E} \setminus \mathbb{S})$ events the outgoing $G.\mathsf{hb}$ edges are only $G.\mathsf{po}$ edges.

(2) If a write event $w \in (G.\mathsf{E} \setminus \mathbb{S})$ is mapped to some promise message, that is, $\mathbb{W}(w) \neq \perp$, then w can have outgoing $G.\mathsf{rfe}$ and mo edges.

Now we state and prove Lemma 3 which use in further proofs.

Lemma 3. .

Given a program $\mathbb{P}$, suppose MS is a promise machine state and G is an WEAKEST *event structure such that G simulates MS; $G \sim \mathsf{MS}$. Then,*

(1) *if two events $a, b \in \mathbb{EW}$ on the same memory location are related by $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ relation in G, then $\mathsf{ts}(a) \leq \mathsf{ts}(b)$. Moreover, if b is a write event then $\mathsf{ts}(a) < \mathsf{ts}(b)$.*

(2) *if two events $a, b \in \mathbb{S}$ on the same memory location are related by $(\mathsf{shb}; \mathsf{seco}^?)$, then $\mathsf{ts}(a) \leq \mathsf{ts}(b)$. Moreover, if b is a write event then $\mathsf{ts}(a) < \mathsf{ts}(b)$.*

(3) *If r reads from w such that $(w, r) \in (G.\mathsf{ew}; G.\mathsf{jf})$ holds then w and r are not hb related, that is $(w, r) \notin (G.\mathsf{hb} \cup G.\mathsf{hb}^{-1})$.*

(4) *Whenever $\mathsf{imm}(\mathsf{spo})(a, b)$ does not hold, $(a, b) \in [G.\mathcal{F}_{SC} \cap \mathbb{S}] ; \mathsf{shb} \cup \mathsf{shb} ; \mathsf{seco} ; \mathsf{shb} ; [G.\mathcal{F}_{SC} \cap \mathbb{S}]$ implies $\mathsf{MS}_a.\mathcal{S} < \mathsf{MS}_b.\mathcal{S}$.*

Proof. We study the component relations of $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ and $(\mathsf{shb}; \mathsf{seco}^?)$.

- **case** $(a, b) \in G.\mathsf{po}_x$
  Let a and b be in the i^{th}thread in the event structure.
  In that case $\mathsf{ts}(a) = MS_a.\mathcal{TS}(i).V(x)$ and $\mathsf{ts}(b) = MS_b.\mathcal{TS}(i).V(x)$.
  We know that promise machine always extends thread view on each location.
  Hence $\mathsf{MS}_a.\mathcal{TS}(i).V(x) \leq \mathsf{MS}_b.\mathcal{TS}(i).V(x)$.
  As a result, $\mathsf{ts}(\mathsf{op}_a) \leq \mathsf{ts}(\mathsf{op}_b)$.
- **case** $(a, b) \in G.\mathsf{rf}$.
  In this case op_a creates the message $\langle x : -@t \rangle$ and op_b reads from the same message in the promise machine. As a result, $\mathsf{ts}(a) = \mathsf{ts}(b)$.
- **case** $(a, b) \in G.\mathsf{ew}$.
  We create $G.\mathsf{ew}$ for the event pairs corresponding to the promise and fulfill operations. In this case op_a, op_b are promise and fulfill operations respectively. The promise operation append a message and the fulfill operation removes the same message from the message queue. Hence, $\mathsf{ts}(a) = \mathsf{ts}(b)$.
- **case** $(a, b) \in G.\mathsf{rf}$.
  We know that
  $G.\mathsf{jf}(a, b) \implies (\mathsf{ts}(a) = \mathsf{ts}(b))$,
  $G.\mathsf{ew}(a, b) \implies (\mathsf{ts}(a) = \mathsf{ts}(b))$, and
  $G.\mathsf{rf} = G.\mathsf{ew}^?; G.\mathsf{jf}$.
  As a result, $G.\mathsf{rf}(a, b) \implies (\mathsf{ts}(a) = \mathsf{ts}(b))$.
- **case** $(a, b) \in G.\mathsf{hb}$.
  In this case $(a, b) \in (G.\mathsf{po} \cup G.\mathsf{sw})^+$.
  If $G.\mathsf{po}(a, b)$ then $(a, b) \in G.\mathsf{po}_x$ and hence $\mathsf{ts}(a) < \mathsf{ts}(b)$.
  Otherwise there exists some event c and d such that $(a, c) \in G.\mathsf{po} \wedge (c, d) \in G.\mathsf{sw} \wedge G.\mathsf{hb}^?(c, b)$.
  Following the promising semantics $\mathsf{ts}(a) \leq \mathsf{MS}_c.\mathcal{TS}(c.\mathsf{tid}).V(x)$.
  Then considering c and d access types
  – $c \in G.\mathcal{F}_{\mathrm{REL}} \cap [\mathsf{Rel}]$ and $d \in G.\mathcal{R} \cap [\mathsf{Acq}]$

In this case there exists some event $w \in \mathbb{EW}$ such that $G.\mathrm{po}(c, w)$, $w.\mathrm{loc} = d.\mathrm{loc}$, $w \in G.\mathcal{W}_{\mathrm{RLX}}$, and $(w, d) \in G.\mathrm{jf}^+$. and op_w results in message $m = \langle - : -@-, R\rangle$.
In this case view $\mathrm{MS}_a.\mathcal{TS}(a.\mathrm{tid}).V(x)$ is included in the message view $m.R$,
that is, $\mathrm{MS}_a.\mathcal{TS}(a.\mathrm{tid}).V(x) \in m.R$.
Now if $G.\mathrm{jf}(w, d)$ then $m.R \in \mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).\mathrm{cur}$
and hence $\mathrm{MS}_a.\mathcal{TS}(a.\mathrm{tid}).V(x) \in m.R \in \mathrm{MS}_b.\mathcal{TS}(b.\mathrm{tid}).\mathrm{cur}$.
Otherwise if $G.\mathrm{jf}(w, u_1) \wedge G.\mathrm{jf}(u_1, u_2) \wedge \ldots \wedge G.\mathrm{jf}(u_n, d)$ where $u_1, u_2, \ldots u_n \in (G.\mathrm{U} \cap \mathbb{EW})$ then following the promising semantics
(i) if $w.\mathrm{loc} \neq c.\mathrm{loc}$ then the view $\mathrm{MS}_a.\mathcal{TS}(a.\mathrm{tid}).V(x)$ propagates through the messages created by $u_1, u_2, \ldots u_n$ and finally reaches d,
that is, $\mathrm{MS}_a.\mathcal{TS}(a.\mathrm{tid}).V(x) \in m.R \in \mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).\mathrm{cur}$ holds.
(ii) if $w.\mathrm{loc} = c.\mathrm{loc}$ then $G.\mathrm{po}_x(c, w)$ and hence $\mathrm{ts}(c) < \mathrm{ts}(w)$
and in consequence $\mathrm{ts}(c) < \mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).V(x)$.
Hence, considering (i) and (ii), $\mathrm{MS}_c.\mathcal{TS}(c.\mathrm{tid}).V(x) \leq \mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).V(x)$ holds.

– $c \in G.\mathcal{W} \cap [\mathrm{Rel}]$ and $d \in G.\mathcal{R} \cap [\mathrm{Acq}]$
Similarly to above, the view $\mathrm{MS}_c.\mathcal{TS}(c.\mathrm{tid}).V(x)$ propagates to $\mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).\mathrm{cur}$ by a read-from or release sequence and in that case
$\mathrm{MS}_c.\mathcal{TS}(c.\mathrm{tid}).V(x) \leq \mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).V(x)$.
– $c \in G.\mathcal{F} \cap [\mathrm{Rel}]$ and $d \in G.\mathcal{F} \cap [\mathrm{Acq}]$
In this case there exists some event $w, r \in \mathbb{EW}$ such that
$G.\mathrm{po}(c, w)$, $w \in G.\mathcal{W}_{\mathrm{RLX}}$, $G.\mathrm{po}(r, d)$, $r \in G.\mathcal{R}_{\mathrm{RLX}}$, and $(w, r) \in G.\mathrm{jf}^+$.
Note that since a fence d is in $\mathbb{EW}$, the $G.\mathrm{po}$-predecessor r is also in $\mathbb{EW}$.
Similar to the earlier case $\mathrm{MS}_c.\mathcal{TS}(c.\mathrm{tid}).V(x)$ propagates to r
and gets included in $\mathrm{MS}_r.\mathcal{TS}(r.\mathrm{tid}).V.\mathrm{acq}$.
Finally $\mathrm{MS}_r.\mathcal{TS}(k).V.\mathrm{acq}$ is included in $\mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).\mathrm{cur}$
and in turn $\mathrm{MS}_c.\mathcal{TS}(c.\mathrm{tid}).V(x) \leq \mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).V(x)$.
– $c \in G.\mathcal{W} \cap [\mathrm{Rel}]$ and $d \in G.\mathcal{F} \cap [\mathrm{Acq}]$
Similar to the earlier case $\mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).\mathrm{cur}$ gets the $\mathrm{MS}_c.\mathcal{TS}(i).V(x)$ or an updated view of x and as a result, $\mathrm{MS}_c.\mathcal{TS}(c.\mathrm{tid}).V(x) \leq \mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).V(x)$.

As a result, $\mathrm{ts}(a) \leq \mathrm{MS}_d.\mathcal{TS}(d.\mathrm{tid}).V(x)$ and following the $G.\mathrm{hb}$ path $\mathrm{ts}(a) \leq \mathrm{ts}(b)$.
In all these $G.\mathrm{hb}$ cases the $\mathrm{ts}(a)$ propagates to b. If b is a write event then it extends the view and updates with a new timestamp. Hence if b is a write then $\mathrm{ts}(a) < \mathrm{ts}(b)$.
Following from this argument, if $(a, b) \in G.\mathrm{mo}_{\mathrm{strong}}$ then $\mathrm{ts}(a) < \mathrm{ts}(b)$ holds.

- $(a, b) \in G.\mathrm{fr}_{\mathrm{strong}}$.
There exists a write c such that $(a, c) \in G.\mathrm{rf}^{-1} \wedge (c, b) \in G.\mathrm{mo}_{\mathrm{strong}}$.
In this case $\mathrm{ts}(a) = \mathrm{ts}(c)$ and $\mathrm{ts}(c) < \mathrm{ts}(b)$ holds.
As a result, $\mathrm{ts}(a) < \mathrm{ts}(b)$ holds.

Thus considering the component relations of $(G.\mathrm{hb}; G.\mathrm{eco}^?_{\mathrm{strong}})|_{\mathrm{loc}}$ results in $\leq$-order following the timestamps of the corresponding promise machine. (1)

We now study the component relations of $(\mathrm{shb}; \mathrm{seco}^?)$.

- $(a, b) \in \mathrm{shb}$
Considering the definition, in this case, $\mathrm{shb} \subseteq G.\mathrm{hb} \cap (\mathbb{EW} \times \mathbb{EW})$.
Hence $\mathrm{shb}(a, b)$ implies $\mathrm{ts}(a) \leq \mathrm{ts}(b)$ and if b is a write event then $\mathrm{ts}(a) < \mathrm{ts}(b)$.
- $(a, b) \in \mathrm{srf}$.
Considering the definition, in this case, $\mathrm{srf} \subseteq G.\mathrm{rf} \cap (\mathbb{EW} \times \mathbb{EW})$. Hence $\mathrm{srf}(a, b)$ implies $\mathrm{ts}(a) = \mathrm{ts}(b)$

- $(a, b) \in \text{smo}$.
  We know $\text{smo} \subseteq \text{mo}$ and hence following the definition of mo, $\text{smo}(a, b)$ implies $\text{ts}(a) < \text{ts}(b)$.
- $(a, b) \in \text{sfr}$.
  Hence $(a, b) \in (\text{srf}^{-1}; \text{smo})$. As a result, $\text{ts}(a) < \text{ts}(b)$.

Thus considering the component relations of $(\text{shb}; \text{seco}^?)|_{\text{loc}}$ results in $\leq$-order following the timestamps of the corresponding promise machine. Moreover, when $(a, b) \in (\text{shb}; \text{seco}^?)|_{\text{loc}}$ and b is a write then $\text{ts}(a) < \text{ts}(b)$. (2)

We now study the relation between w and r when $(w, r) \in (G.\text{ew}; G.\text{jf})$.
We consider two cases

- **case** $G'.\text{hb}(w, r)$ does not hold as $w.\text{ord} \sqsubset \text{REL}$.
- **case** $G'.\text{hb}(r, w)$.
  From (1), in this case $G'.\text{hb}(r, w)$ implies $\text{ts}(r) < \text{ts}(w)$. However, we know, $G.\text{rf}(w, r)$ implies $\text{ts}(r) = \text{ts}(w)$.
  Hence a contradiction and $G'.\text{hb}(r, w)$ does not hold.

As a result, $(w, r) \notin (G.\text{hb} \cup G.\text{hb}^{-1})$. (3)

We have to show that $(a, b) \in [G.\mathcal{F}_{\text{SC}} \cap \mathbb{S}]; \text{shb} \cup \text{shb}; \text{seco}; \text{shb}; [G.\mathcal{F}_{\text{SC}} \cap \mathbb{S}]$ implies $\text{MS}_a.\mathcal{S} \leq \text{MS}_b.\mathcal{S}$.

When $\text{shb}(a, b)$, then either the SC view $\text{MS}_a.\mathcal{S}$ propagates to MS_b or is overwritten by intermediate greater timestamps on the locations. $\text{MS}_a.\mathcal{S} = \text{MS}_b.\mathcal{S}$ holds only when two consecutive SC fences are executed, that is, $\text{imm}(G.\text{po})(a, b)$ holds.

Otherwise, similar to (1) we can perform case analysis on the shb path and
show that $\text{MS}_a.\mathcal{S}_x < \text{MS}_b.\mathcal{S}_x$ for at least one location $x \in \text{Locs}$.

When $(a, b) \in (\text{shb}; \text{seco}; \text{shb})$ then let there are intermediate event $c, d \in \mathbb{EW}$ such that $\text{shb}(a, c)$, $\text{seco}(c, d)$, and $\text{shb}(d, b)$ holds. In this case $\text{MS}_a.\mathcal{S} < \text{MS}_c.\mathcal{TS}(c.\text{tid}).V$.

From the similar argument as (2), we can show that the timestamps increase or remain same through seco edges from c to d on location $c.\text{loc}$.

Hence $\text{seco}(c, d)$ implies $\text{MS}_c.\mathcal{TS}(c.\text{tid}).V < \text{MS}_d.\mathcal{TS}(d.\text{tid}).V$ and
$\text{shb}(d, b)$ implies $\text{MS}_d.\mathcal{TS}(d.\text{tid}).V \leq \text{MS}_b.\mathcal{S}$.

As a result, whenever $\text{imm}(\text{spo})(a, b)$ does not hold,
$(a, b) \in [G.\mathcal{F}_{\text{SC}} \cap \mathbb{S}]; \text{shb} \cup \text{shb}; \text{seco}; \text{shb}; [G.\mathcal{F}_{\text{SC}} \cap \mathbb{S}]$ implies $\text{MS}_a.\mathcal{S} < \text{MS}_b.\mathcal{S}$. □

Lemma 4. *Given a program $\mathbb{P}$, suppose MS is a promise machine state and G is an* WEAKEST *event structure such that G simulates MS; $G \sim \text{MS}$. In this case there is no outgoing external-synchronization from $G.\text{E} \setminus \mathbb{S}$ events, that is,* $\text{dom}(G.\text{swe}) \subseteq \mathbb{S}$.

Proof. The simulation construction steps ensure that the conflicting events of $\mathbb{S}$, that is, $G.\text{E} \setminus \mathbb{S}$ events are created only as part of PS certificate steps in the respective threads.

In the promising semantics the certificate steps are not visible to any other thread. Similarly in event structure G the there is no outgoing rfe edge from $G.\text{E} \setminus \mathbb{S}$ events except the event corresponding to the promise. Let that event be e_p.

From PS we know that $e_p.\text{ord} \sqsubseteq \text{RLX}$ and certificate steps do not have any release fence. Hence $G.\mathcal{F}_{\sqsupseteq\text{REL}} \cap (G.\text{E} \setminus \mathbb{S}) = \emptyset$.

Hence there is no outgoing $G.\text{swe}$ edge from $G.\text{E} \setminus \mathbb{S}$ events and $\text{dom}(G.\text{swe}) \subseteq \mathbb{S}$ holds. □

Next we restate and prove Lemma 1.

Lemma 1. $G \sim_{\{i\}} \text{MS} \wedge \text{MS} \xrightarrow{np}_i \text{MS}' \implies \exists G'.\ G \rightarrow_{\mathbb{P},\textit{WEAKEST}}{}^* G' \wedge G' \sim_{\{i\}} \text{MS}'$.

Before going to the proof we restate the proof idea.

Proof Idea. The G' is constructed in two steps.

(1) First, for a non-promise operation np we either append a corresponding event e' to G or we identify an existing corresponding event e' in G. In earlier case G is extended to G' and in later case $G' = G$.

(2) Next, we check whether TS_i has outstanding promises. If so, then we know that there is a promise-free certificate which fulfills the outstanding promises. In that case, for each non-promise certificate step we extend the event structure following the rules in WEAKEST and at each step the constructed event structure remains consistent.

In this construction G and MS are related by $\mathbb{S}$, $\mathbb{W}$, and we define $\mathbb{S}'$, $\mathbb{W}'$ to relate the G' and MS$'$. By using the definitions of $\mathbb{S}'$, $\mathbb{W}'$ we show that $G' \sim_{\{i\}}$ MS$'$ holds. We use the results of Lemma 3 to establish the simulation relation.

Proof. We do a case analysis on the operation op of the promise machine transition $\mathsf{MS} \xrightarrow{np}_i \mathsf{MS}'$ where op $= np$. From the definition of the simulation relation we know $\forall i.\ \mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$. Hence we can also make a step from the event structure G to G'.

Case Store $\mathsf{St}(o, x, v)$ creating message m':

In the event structure we extend the event structure G to G'. We extend the cover set $\mathbb{S}_i$ as well as the relations (spo, srf, smo) to $\mathbb{S}'_i$ along with the respective relations (spo$'$, srf$'$, smo$'$) by including an event e' where

(1) $\mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,

(2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and

(3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e'.\mathsf{lab}) \in \mathbb{P}(i)$.

In this case the promise machine is updated as follows.

$\mathsf{M}' = \mathsf{M} \uplus \{m'\}$, $\mathcal{S}' = \mathcal{S}$, and

$\mathcal{TS}' = \mathcal{TS}[i \mapsto \langle\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle, V', \mathcal{TS}(i).\mathsf{P}\rangle]$ where $V' = \mathcal{TS}(i).V[x \mapsto m'.ts]$.

Now we do a case analysis on whether such a store event e' exists in G or we append a new event.

Subcase $\nexists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\mathsf{lab} = \mathsf{St}_o(x, v)$:

We create e' such that $e'.\mathsf{lab} = \mathsf{St}_o(x, v)$ and append e' to event structure G to create G'. Then,

- $G'.\mathsf{E} = G.\mathsf{E} \uplus \{e'\}$
- $G'.\mathsf{po} = (G.\mathsf{po} \cup \{(e, e') \mid e \in (\mathbb{S}_i \cup \mathbb{S}_0)\})^+$
- $G'.\mathsf{jf} = G.\mathsf{jf}$
- $G'.\mathsf{ew} = G.\mathsf{ew}$

Let: $\mathbb{W}' \triangleq \mathbb{W}[e' \mapsto m']$.

Based on $\mathbb{W}'$, we derive following definitions in MS$'$.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathsf{mo}' \triangleq \mathsf{mo} \uplus \{(a, e') \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(a).ts < \mathbb{W}'(e').ts\}$
  $\uplus \{(e', a) \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(e').ts < \mathbb{W}'(a).ts\}$
- $\mathsf{sc}' \triangleq \mathsf{sc}$
- $\mathsf{spo}' \triangleq (\mathsf{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathsf{srf}' \triangleq \mathsf{srf}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: *G' is consistent in* WEAKEST *model.*

- (CF) We know that G satisfies constraint (CF). Considering the definition of G'.ecf, the only incoming hb edge is G'.po and there is no outgoing edge from event e'. Hence G'.ecf is irreflexive and G' satisfies (CF).
- (CFJ) We know that G satisfies constraint (CFJ). We also know that $G'.\mathsf{jf} = G.\mathsf{jf}$ and event e' has no outgoing G'.hb or G'.jf edge. Hence $G'.\mathsf{jf} \cap G'.\mathsf{ecf} = \emptyset$ and G' satisfies (CFJ).
- (VISJ) Constraint (VISJ) is preserved in G' as $G'.\mathsf{jf} = G.\mathsf{jf}$ and G satisfies constraint (VISJ).
- (ICF) We know that G satisfies (ICF). Suppose there exists an event $e_1 \in G$ which is in immediate conflict with e' in G', that is $G'. \sim (e_1, e')$ holds.
  Then (1) $\mathrm{dom}(G.\mathrm{po}; [\{e_1\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e_1 \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathrm{sequence}_{G.\mathrm{po}}(\mathbb{S}_i)).(e_1.\mathrm{lab}) \in \mathbb{P}(i)$.
  However, from definition of e' we already know that
  (1) $\mathrm{dom}(G.\mathrm{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathrm{sequence}_{G.\mathrm{po}}(\mathbb{S}_i)).(e'.\mathrm{lab}) \in \mathbb{P}(i)$.
  Hence following the determinacy condition we know either $e_1 = e'$ or there exists no such e_1. Hence (ICF) is preserved in G'.
- (ICFJ) Constraint (ICFJ) is preserved in G' as $e' \notin \mathcal{R}$ and G satisfies constraint (ICFJ).
- (COH) We know G preserves (COH) constraint, that is, $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathrm{strong}})$ is acyclic. The incoming edges to event e' are G'.po, $G'.\mathsf{fr}_{\mathrm{strong}}$, G'.hb and there is no outgoing edge concerning G'.hb or $G'.\mathsf{eco}_{\mathrm{strong}}$. As a result, $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathrm{strong}})$ is acyclic and G' preserves (COH) constraint.

(2) Condition to show: *The local state of each thread in* MS′ *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*

In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathrm{sequence}_{\mathrm{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
case For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS′ and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
case For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathrm{sequence}_{\mathrm{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo′ we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathrm{sequence}_{\mathrm{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathrm{sequence}_{\mathrm{spo}}(\mathbb{S}_i)) \cdot e'.\mathrm{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma \cdot e'.\mathrm{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS′ and G'.

(3) Condition to show: *Whenever* $\mathbb{W}'$ *maps an event of* G' *to a message in* MS′, *then the location accessed and the written values match.*

We know that the event to message mappings for existing events in G.E and messages M do not change.

$$\forall e \in G'.\mathsf{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$$

If $e = e'$ then $\mathbb{W}'(e') = m'$ and $e'.\mathrm{loc} = m'.\mathrm{loc} = x$ and $e'.\mathrm{wval} = m'.\mathrm{wval} = v$.
Hence $\mathbb{W}'$ preserves the condition.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$, *there are corresponding write events in* G' *that are* po-*after* $\mathbb{S}'$.

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS$'$, that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

(5) Condition to show: *For every location ℓ and thread j, the thread view of ℓ in the promise state* MS$'$ *records the timestamp of the maximal write visible to the covered events in G' of thread j.*
Essentially we have to show
$\forall j, \ell.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_j])\}$
case For $j \neq i$ or $j = i \wedge \ell \neq x$, it is trivial because $\mathcal{TS}'.V(\ell) = \mathcal{TS}.V(\ell)$.
case For $j = i \wedge \ell = x$,
following the promising semantics $e' \in G.\mathcal{W}_x$, $\mathbb{W}'(e') = m'$, $m'.ts$ extends the view on x in thread i, and hence $\mathcal{TS}(i).V(x) < \mathcal{TS}'(i).V(x)$.
In this case $e' \in \mathbb{S}'_i$ and hence $e' \in \mathsf{dom}([\mathcal{W}_x]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])$ holds.
As a result,
$\mathcal{TS}'(i).V(x) = m'.ts = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$.
Thus the relation holds between MS$'$ and G'.

(6) Condition to show: *The $\mathbb{S}'$ events in G' preserve coherence: $\mathsf{shb}'; \mathsf{seco}'^?$ is irreflexive.*
We know $e' \in \mathbb{S}'$ and let $a \in \mathbb{S}'$ such that $(a, e') \in (\mathsf{shb}'; \mathsf{seco}'^?)$.
Hence following the definitions of shb', seco', and from Lemma 3 (2)
we know $\mathsf{MS}'_a.\mathcal{TS}'(a.\mathsf{tid}).V(x) < \mathsf{MS}_{e'}.\mathcal{TS}'(e'.\mathsf{tid}).V(x)$ as $e' \in \mathsf{St}$.
As a result, $(\mathsf{shb}'; \mathsf{seco}'^?)$ is irreflexive.

(7) Condition to show: *The atomicity condition for update operations holds for $\mathbb{S}'$ events in G'.*
We know that $[G'.\mathsf{U} \cap \mathbb{S}'] = [G.\mathsf{U} \cap \mathbb{S}]$ and $[G.\mathsf{U} \cap \mathbb{S}]; (\mathsf{sfr}; \mathsf{smo}) = \emptyset$ holds.
Assume there exists an update $u \in G'.\mathsf{U} \cap \mathbb{S}'$, which reads from w, such that $\mathsf{sfr}'(u, e')$ and $\mathsf{smo}'(e', u)$ holds.
By the definitions of sfr' and smo', $\mathbb{W}'(w).ts < m'.ts < \mathbb{W}'(u).ts$.
But the promising semantics does not assign a timestamp in that range.
Hence a contradiction and $[G'.\mathsf{U} \cap \mathbb{S}']; (\mathsf{sfr}'; \mathsf{smo}') = \emptyset$ holds.

(8) Condition to show: *The SC fences in G' are appropriately ordered by sc'.*
We know $[G.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}$ holds in G.
From definitions we know, $G'.\mathcal{F}_{\mathsf{sc}} = G.\mathcal{F}_{\mathsf{sc}}$, $\mathsf{sc}' = \mathsf{sc}$, $\mathsf{shb} \subseteq \mathsf{shb}'$, $\mathsf{seco} \subseteq \mathsf{seco}'$.
Consider a, b are two SC fences such that $(a, b) \in [G.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathcal{F}_{\mathsf{sc}}]$, and $\mathsf{sc}(a, b)$ holds.
In that case $(a, b) \in (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$ holds and $\mathsf{sc}'(a, b)$ holds.
To show $[G'.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$, we have to show $(b, a) \notin (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$. We show this by contradiction.
Assume $(b, a) \in (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
This is possible due to the relations created to/from event e'.
Considering the relations in shb' and seco', the incoming relations to event e' are shb', sfr', smo' and the outgoing edges are smo'.
As there is no outgoing srf edge from e', no new synchronization edge is created, that is, $\mathsf{ssw}' = \mathsf{ssw}$.
Thus a $\mathsf{smo}'(e', w)$ edge where w is a write event occurs in the $(\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$ path from b to a.
In this case the path from b to a is $(b, e') \in \mathsf{shb}'; \mathsf{seco}'^?$ and $(e', a) \in \mathsf{smo}'; \mathsf{seco}'^?; \mathsf{shb}'$.
We analyze the cases of $(b, e') \in \mathsf{shb}'; \mathsf{seco}'^?$.

- **case** $\text{shb}'(b, e')$.
  In this case $\text{shb}(b, e)$ and $\text{spo}'(e, e')$ hold.
  Hence $\text{MS}_b.\mathcal{TS}(b.\text{tid}).V(x) \leq \text{MS}_e.\mathcal{TS}(e.\text{tid}).V(x) < \text{MS}_{e'}.\mathcal{TS}(e'.\text{tid}).V(x)$.
- **case** $\text{shb}'; \text{seco}'(b, c)$ and $\text{smo}'(c, e')$.
  Hence $\text{shb}; \text{seco}(b, c)$ and $\text{smo}'(c, e')$ holds.
  So $\text{MS}_b.\mathcal{TS}(b.\text{tid}).V(x) \leq \text{MS}_c.\mathcal{TS}(c.\text{tid}).V(x) < \text{MS}_{e'}.\mathcal{TS}(e'.\text{tid}).V(x)$.

Now we analyze $(e', a) \in \text{smo}'; \text{seco}'^?; \text{shb}'$.
In this case there exist a write $w \in \mathbb{S}$ such that
$\text{smo}'(e', w)$ and $(w, a) \in \text{seco}^?; \text{shb}$ holds.
Hence $\text{MS}_{e'}.\mathcal{TS}(e'.\text{tid}).V(x) < \text{MS}_w.\mathcal{TS}(w.\text{tid}).V(x) \leq \text{MS}_a.\mathcal{TS}(a.\text{tid}).V(x)$.
As a result, in all cases $\text{MS}_b.\mathcal{TS}(b.\text{tid}).V(x) < \text{MS}_a.\mathcal{TS}(a.\text{tid}).V(x)$ holds.
However, we know that $\text{sc}(a, b)$ holds and therefore we have
$\text{MS}_a.\mathcal{TS}(a.\text{tid}).V(x) \leq \text{MS}_b.\mathcal{TS}(b.\text{tid}).V(x)$.
This is a contradiction and hence $(b, a) \notin (\text{shb}' \cup \text{shb}'; \text{seco}'; \text{shb}')$.
As a result, $[G'.\mathcal{F}_{\text{sc}}]; \text{shb}' \cup \text{shb}'; \text{seco}'; \text{shb}'; [G'.\mathcal{F}_{\text{sc}}] \subseteq \text{sc}'$ holds.

(9) Condition to show: *The behavior of* MS′ *matches that of the* $\mathbb{S}'$ *events in* G'.
Essentially we have to show, $\text{Behavior}(\text{MS}') = \text{Behavior}(G', \mathbb{W}', \mathbb{S}')$.
Following the definitions of $\text{Behavior}(\text{MS}')$ and $\text{Behavior}(G', \mathbb{W}', \mathbb{S}')$; we know following cases for a location ℓ:

- **case** $\ell \neq x$:
  The set of messages on $\ell \neq x$ remains from MS to MS′.
  Hence in the promise machine $\text{Behavior}|_\ell(\text{MS}') = \text{Behavior}|_\ell(\text{MS})$ holds.
  Similarly $\text{Behavior}|_\ell(G', \mathbb{W}', \mathbb{S}') = \text{Behavior}|_\ell(G, \mathbb{W}, \mathbb{S})$ holds in the event structure.
  We already know that $\text{Behavior}|_\ell(\text{MS}) \subseteq \text{Behavior}|_\ell(G, \mathbb{W}, \mathbb{S})$ holds for MS and G.
  As a result, $\text{Behavior}|_\ell(\text{MS}') = \text{Behavior}|_\ell(G', \mathbb{W}', \mathbb{S}')$.
- **case** $\ell = x$:
  Let m be the message on x which results in the behavior of MS. In that case $m.\text{loc} = x$, $\text{maxmsg}(\text{M} \setminus \bigcup_i \mathcal{TS}(i).\text{P}, x) = m$, and let $m.\text{wval} = v_1$. As a result, $(x, v_1) \in \text{Behavior}(\text{MS})$.
  In this case there exists event $e_1 \in G.\mathcal{W}_x \cap \mathbb{S}$ such that $\mathbb{W}(e_1) = m$, $e_1.\text{loc} = x$, $e_1.\text{wval} = v_1$, and $\nexists e_2 \mathbb{S}.\ \text{mo}(e_1, e_2)$.
  Considering the new message is m', we know $m' = \mathbb{W}'(e')$ and $m'.\text{wval} = v$ holds.
  Comparing the m and m' we have two subcases:
  - **subcase** $m.ts < m'.ts$.
    In this case $\text{maxmsg}(\text{M}' \setminus \bigcup_i \mathcal{TS}'(i).\text{P}, x) = m'$ and $\text{Behavior}|_x(\text{MS}') = \{(x, v)\}$.
    In the event structure G', $\text{mo}'(e_1, e')$ holds and hence $\text{Behavior}|_x(G', \mathbb{W}', \mathbb{S}') = \{(x, v)\}$.
  - **subcase** $m.ts > m'.ts$.
    In this case $\text{maxmsg}(\text{M}' \setminus \bigcup_i \mathcal{TS}'(i).\text{P}, x) = \text{maxmsg}(\text{M} \setminus \bigcup_i \mathcal{TS}(i).\text{P}, x)$
    and $\text{Behavior}|_x(\text{MS}') = \text{Behavior}|_x(\text{MS}) = \{(x, v_1)\}$.
    In the event structure $\text{mo}'(e', e_1)$ holds and hence
    $\text{Behavior}|_x(G', \mathbb{W}', \mathbb{S}') = \text{Behavior}|_x(G, \mathbb{W}, \mathbb{S}) = \{(x, v_1)\}$.

  In both cases $\text{Behavior}|_x(G', \mathbb{W}', \mathbb{S}') = \text{Behavior}|_x(\text{MS}')$ holds.

As a result, $\text{Behavior}(G', \mathbb{W}', \mathbb{S}') = \text{Behavior}(\text{MS}')$.

Subcase $\exists e' \in (G.\text{E}_i \setminus \mathbb{S}_i).\ \text{dom}(G.\text{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\text{lab} = \text{St}_o(x, v)$**:**
We take $G' = G$ and let $\mathbb{W}' \triangleq \mathbb{W}[e' \mapsto m']$.
Based on $\mathbb{W}'$, we derive following definitions in MS′.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$

- $\text{mo}' \triangleq \text{mo} \uplus \{(a, e') \mid a \in G.\mathcal{W}_x \cap \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(a).ts < \mathbb{W}'(e').ts\}$
  $\uplus \{(e', a) \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(e').ts < \mathbb{W}'(a).ts\}$
- $\text{sc}' \triangleq \text{sc}$
- $\text{spo}' \triangleq (\text{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\text{srf}' \triangleq \text{srf}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: *G' is consistent in* WEAKEST *model.*

G' is consistent as G is consistent.

(2) Condition to show: *The local state of each thread in* MS$'$ *contains the program of that thread along with the sequence of covered events in G' of that thread.*

In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \textit{labels}(\text{sequence}_{\text{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
case For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS$'$ and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
case For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \textit{labels}(\text{sequence}_{\text{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo$'$ we get
$\langle \mathbb{P}(i), \textit{labels}(\text{sequence}_{\text{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \textit{labels}(\text{sequence}_{\text{spo}}(\mathbb{S}_i)) \cdot e'.\text{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma \cdot e'.\text{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS$'$ and G'.

Note. This was same as the other scenario when we append a new $\text{St}_o(x, v)$.

(3) Condition to show: *Whenever $\mathbb{W}'$ maps an event of G' to a message in* MS$'$*, then the location accessed and the written values match.*

case The event to message mappings for existing events in G.E and messages M do not change. Hence $\forall e \in G'.\text{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$.
If $e = e'$ then $\mathbb{W}'(e') = \text{wmsg}(\text{op}) = m'$ and $e'.\text{loc} = \text{wmsg}(\text{op}).\text{loc} = x$ and $e.\text{wval} = m'.\text{wval} = v$.

Thus $\mathbb{W}'$ preserves the condition between MS$'$ and G'.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$*, there are corresponding write events in G' that are* po*-after* $\mathbb{S}'$.

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS$'$, that is, $\forall j \neq i.\ \mathcal{TS}(j).\text{P} = \mathcal{TS}'(j).\text{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

Note. This was same as the other scenario when we append a new $\text{St}_o(x, v)$.

(5) Condition to show: *For every location ℓ and thread j, the thread view of ℓ in the promise state* MS$'$ *records the timestamp of the maximal write visible to the covered events in G' of thread j.*

Essentially we have to show
$\forall j, \ell.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \text{dom}([\mathcal{W}_\ell]; G'.\text{jf}^?; \text{shb}'^?; \text{sc}'^?; \text{shb}'^?; [\mathbb{S}'_j])\}$
For $j \neq i$ or $j = i \wedge \ell \neq x$, it is trivial because $\mathcal{TS}'.V(\ell) = \mathcal{TS}.V(\ell)$.
For $j = i \wedge \ell = x$, from the definition we know
(1) $\mathcal{TS}(i).V(x) = \max\{\mathbb{W}(e).ts \mid e \in \text{dom}([\mathcal{W}_x]; G.\text{jf}^?; \text{shb}^?; \text{sc}^?; \text{shb}^?; [\mathbb{S}_i])\}$
(2) $\mathcal{TS}'(i).V(x) = m'.ts$

(3) $\mathbb{W}'(e') = m'$ holds.
Following the promising semantics, we know $\mathcal{TS}'(i).V(x)$ extends the thread view of x from $\mathcal{TS}(i).V(x)$ and $\mathcal{TS}(i).V(x) < m'.ts$.
Hence following the construction,
$\mathcal{TS}'(i).V(x) = m'.ts = \max\{\mathbb{W}'(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_x]; G'.\mathrm{jf}^?; \mathrm{shb}'^?; \mathrm{sc}'^?; \mathrm{shb}'^?; [\mathbb{S}'_i])\}$ holds.
Thus the relation holds between MS' and G'.

(6) Condition to show: *The $\mathbb{S}'$ events in G' preserve coherence:* $\mathrm{shb}'; \mathrm{seco}'^?$ *is irreflexive.*
The argument is analogous to the case when we append a new $\mathrm{St}_o(x, v)$.

(7) Condition to show: *The atomicity condition for update operations holds for $\mathbb{S}'$ events in G'.*
The argument is analogous to the case when we append a new $\mathrm{St}_o(x, v)$.

(8) Condition to show: *The* SC *fences in G' are appropriately ordered by* sc'.
The argument is analogous to the case when we append a new $\mathrm{St}_o(x, v)$.

(9) Condition to show: *The behavior of* MS' *matches that of the $\mathbb{S}'$ events in G'.*
Essentially we have to show, $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$.
Following the definitions of $\mathsf{Behavior}(\mathsf{MS}')$ and $\mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$; we know following cases for a location ℓ:
   - **case** $\ell \neq x$:
     The set of messages on $\ell \neq x$ remains from MS to MS'.
     Hence in the promise machine $\mathsf{Behavior}|_\ell(\mathsf{MS}') = \mathsf{Behavior}|_\ell(\mathsf{MS})$ holds.
     Similarly $\mathsf{Behavior}|_\ell(G', \mathbb{W}', \mathbb{S}') = \mathsf{Behavior}|_\ell(G, \mathbb{W}, \mathbb{S})$ holds in the event structure.
     We already know that $\mathsf{Behavior}|_\ell(\mathsf{MS}) = \mathsf{Behavior}|_\ell(G, \mathbb{W}, \mathbb{S})$ holds for MS and G.
     As a result, $\mathsf{Behavior}|_\ell(\mathsf{MS}') = \mathsf{Behavior}|_\ell(G', \mathbb{W}', \mathbb{S}')$.
   - **case** $\ell = x$:
     Let m be the message on x which results in the behavior of MS. In that case $m.\mathsf{loc} = x$, $\mathsf{maxmsg}(\mathsf{M} \setminus \bigcup_i \mathcal{TS}(i).\mathsf{P}, x) = m$, and let $m.\mathsf{wval} = v_1$. As a result, $(x, v_1) \in \mathsf{Behavior}(\mathsf{MS})$.
     In this case there exists event $e_1 \in G.\mathcal{W}_x \cap \mathbb{S}$ such that $\mathbb{W}(e_1) = m$, $e_1.\mathsf{loc} = x$, $e_1.\mathsf{wval} = v_1$, and $\nexists e_2 \in \mathbb{S}.\ \mathrm{mo}(e_1, e_2)$.
     Considering the new message is m', we know $m' = \mathbb{W}'(e')$ and $m'.\mathsf{wval} = v$ holds.
     Comparing the m and m' we have two subcases:
     – **subcase** $m.ts < m'.ts$.
       In this case $\mathsf{maxmsg}(\mathsf{M}' \setminus \bigcup_i \mathcal{TS}'(i).\mathsf{P}, x) = m'$ and $\mathsf{Behavior}|_x(\mathsf{MS}') = \{(x, v)\}$.
       In the event structure G', $\mathrm{mo}'(e_1, e')$ holds and hence $\mathsf{Behavior}|_x(G', \mathbb{W}', \mathbb{S}') = \{(x, v)\}$.
     – **subcase** $m.ts > m'.ts$.
       In this case $\mathsf{maxmsg}(\mathsf{M}' \setminus \bigcup_i \mathcal{TS}'(i).\mathsf{P}, x) = \mathsf{maxmsg}(\mathsf{M} \setminus \bigcup_i \mathcal{TS}(i).\mathsf{P}, x)$ and $\mathsf{Behavior}|_x(\mathsf{MS}') = \mathsf{Behavior}|_x(\mathsf{MS}) = \{(x, v_1)\}$.
       In the event structure $\mathrm{mo}'(e', e_1)$ holds and hence
       $\mathsf{Behavior}|_x(G', \mathbb{W}', \mathbb{S}') = \mathsf{Behavior}|_x(G, \mathbb{W}, \mathbb{S}) = \{(x, v_1)\}$.
     In both cases $\mathsf{Behavior}|_x(G', \mathbb{W}', \mathbb{S}') = \mathsf{Behavior}|_x(\mathsf{MS}')$ holds.

As a result, $\mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}') = \mathsf{Behavior}(\mathsf{MS}')$.
Note. This was same as the other scenario when we append a new $\mathrm{St}_o(x, v)$.

Case READ $\mathsf{Ld}(o, x, v)$ **reading from message** $\mathsf{wm} = \langle x : v@(-, t], R\rangle$**:**
In the event structure we extend the event structure G to G'. We extend the cover set $\mathbb{S}_i$ as well as the relations $(\mathrm{spo}, \mathrm{srf}, \mathrm{smo})$ to $\mathbb{S}'_i$ along with the respective relations $(\mathrm{spo}', \mathrm{srf}', \mathrm{smo}')$ by including an event e' where
(1) $\mathrm{dom}(G.\mathrm{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,

(2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
(3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e'.\mathsf{lab}) \in \mathbb{P}(i)$.

In this case the promise machine is updated as follows.
$\mathsf{M}' = \mathsf{M}$, $\mathcal{S}' = \mathcal{S}$, and $\mathcal{TS}' = \mathcal{TS}[i \mapsto \langle\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle, V', \mathcal{TS}(i).\mathsf{P}\rangle]$ where $V' = \mathcal{TS}(i).V[x \mapsto \mathsf{wm}.ts]$.

Now we do a case analysis on whether such an load event e' exists in G or we append a new event.

Subcase $\nexists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathsf{dom}(G.\mathsf{po};[\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\mathsf{lab} = \mathsf{Ld}_o(x, v) \wedge G.\mathsf{jf}(w_m, e')$ **where** $\mathsf{wm} = \mathbb{W}(w_m)$**:**

We create e' such that $e'.\mathsf{lab} = \mathsf{Ld}_o(x, v)$ and append e' to event structure G to create G'. In that case

- $G'.\mathsf{E} = G.\mathsf{E} \uplus \{e'\}$
  $G'.\mathsf{po} = (G.\mathsf{po} \cup \{(e, e') \mid e \in (\mathbb{S}_i \cup \mathbb{S}_0)\})^+$
- $G'.\mathsf{jf} = G.\mathsf{jf} \uplus \{(w_m, e') \mid \mathbb{W}(w_m) = \mathsf{wm} \wedge [\mathbb{S}_0 \cup \mathbb{S}'_i]; G'.\mathsf{po}^?; [\{w_m\}]\}$
- $G'.\mathsf{ew} = G.\mathsf{ew}$

Let: $\mathbb{W}' \triangleq \mathbb{W}$.
Based on $\mathbb{W}'$, we derive following definitions in MS'.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathsf{mo}' \triangleq \mathsf{mo}$
- $\mathsf{sc}' \triangleq \mathsf{sc}$
- $\mathsf{spo}' \triangleq (\mathsf{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathsf{srf}' \triangleq \mathsf{srf} \uplus \{(w, e') \mid G'.\mathsf{rf}(w, e') \wedge w \in \mathbb{S}\}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: *G' is consistent in* WEAKEST *model.*

- (CF)
  We know G preserves (CF). Hence in G' we need to only consider the e'.
  Assume there exists event e_1 and e_2 such that
  $G'.\mathsf{hb}(e_1, e')$, $G'.\mathsf{cf}(e_1, e_2)$, $G'.\mathsf{hb}(e_2, e')$ hold.
  assert: $e_1 \in \mathbb{S}$.
  We know $G'.\mathsf{hb}(e_1, e')$.
  Hence either $G'.\mathsf{po}(e_1, e')$ or $(e_1, e') \in G'.\mathsf{po}^?; G'.\mathsf{swe}; G.\mathsf{hb}^?$.
  case $G'.\mathsf{po}(e_1, e')$. From the definitions $e_1 \in \mathbb{S}$.
  case $(e_1, e') \in G'.\mathsf{po}^?; G'.\mathsf{swe}; G.\mathsf{hb}^?$.
  Assume $e_1 \notin \mathbb{S}$ and hence $e_1 \in G.\mathsf{E} \setminus \mathbb{S}$.
  All po-following events of e_1 are in $G.\mathsf{E} \setminus \mathbb{S}$, that is, $\mathsf{codom}([\{e_1\}].G.\mathsf{po}) \in G.\mathsf{E} \setminus \mathbb{S}$.
  However, from Lemma 4 we know that $\mathsf{dom}(G.\mathsf{swe}) \subseteq \mathbb{S}$ and the events in $G.\mathsf{E} \setminus \mathbb{S}$ has no outgoing swe edge, that is, $\mathsf{dom}(G.\mathsf{swe}) \notin (G.\mathsf{E} \setminus \mathbb{S})$.
  Hence a contradiction and $e_1 \in \mathbb{S}$.
  assert: $e_2 \notin \mathbb{S}$.
  Assume $e_2 \in \mathbb{S}$.
  From the definition of $\mathbb{S}$ it is conflict-free, that is, $\mathbb{S} \cap G.\mathsf{cf} = \emptyset$. Thus it is not possible and hence a contradiction.
  As a result, $e_2 \notin \mathbb{S}$.
  Now we know that $G'.\mathsf{hb}(e_2, e')$ hold and thus $(e_2, e') \in G'.\mathsf{po}^?; G'.\mathsf{swe}; G'.\mathsf{hb}^?$.

From Lemma 4 we know that e_2 has no G'.po following event with outgoing G'.swe. Hence G.po(e_2, e') holds.
In that case G'.po(e_1, e'), G'.po(e_2, e'), G'.cf(e_1, e_2) result in a contradiction.
As a result, G satisfies (CF).

- (CFJ) We know G preserves (CFJ). Hence in G' we need to only consider the G'.jf(w_m, e').
Assume there exists event e_1 and e_2 such that
G'.hb(e_1, e'), G'.cf(e_1, e_2), G'.hb(e_2, w_m) hold.
assert: $e_1 \in \mathbb{S}$.
We know G'.hb(e_1, e').
Hence either G'.po(e_1, e') or $(e_1, e') \in G'.\mathrm{po}^?; G'.\mathrm{swe}; G.\mathrm{hb}^?$.
case G'.po(e_1, e'). From the definitions $e_1 \in \mathbb{S}$.
case $(e_1, e') \in G'.\mathrm{po}^?; G'.\mathrm{swe}; G.\mathrm{hb}^?$.
Assume $e_1 \notin \mathbb{S}$ and hence $e_1 \in G.\mathsf{E} \setminus \mathbb{S}$.
In that case all po following events are in $G.\mathsf{E} \setminus \mathbb{S}$, that is, codom$([\{e_1\}].G.\mathrm{po}) \in G.\mathsf{E} \setminus \mathbb{S}$.
However, from Lemma 4 we know that dom$(G.\mathrm{swe}) \subseteq \mathbb{S}$ and the events in $G.\mathsf{E} \setminus \mathbb{S}$ has no outgoing swe edge, that is, dom$(G.\mathrm{swe}) \notin (G.\mathsf{E} \setminus \mathbb{S})$.
Hence a contradiction and $e_1 \in \mathbb{S}$.
assert: $e_2 \notin \mathbb{S}$.
Assume $e_2 \in \mathbb{S}$.
From the definition of $\mathbb{S}$ it is conflict-free, that is, $\mathbb{S} \cap G.\mathrm{cf} = \emptyset$. Thus it is not possible and hence a contradiction.
As a result, $e_2 \notin \mathbb{S}$.
Now we know that G'.hb(e_2, w_m) as well as G.hb(e_2, w_m) hold and
thus $(e_2, w_m) \in G'.\mathrm{po}^?; G'.\mathrm{swe}; G'.\mathrm{hb}^?$.
From Lemma 4 we know that e_2 has no G'.po following event with outgoing G'.swe. Hence G.po(e_2, w_m) holds.
As a result, $e_1.\mathrm{tid} = e_2.\mathrm{tid} = w_m.\mathrm{tid}$ holds.
However, from the definition of G'.jf(w_m, e') we know that G'.po(e_1, w_m) holds.
In that case G'.po(e_1, w_m), G'.po(e_2, w_m), G'.cf(e_1, e_2) result in a contradiction.
As a result, G satisfies (CFJ).
- (VISJ) We study the possible cases of w_m.
  - If G'.po(w_m, e') then the condition holds as $(w_m, e') \notin G'$.jfe.
  - We will show that G' satisfies (CFJ) constraint. Hence w_m cannot be in conflict with e', that is, $(w_m, e') \notin G'$.cf.
  - w_m is in different thread and G'.jfe(w_m, e') holds. We know that $G \sim_{\{i\}}$ MS and the simulation rules ensures that there is no *invisible* event in the $(\mathsf{T} \setminus \{i\})$ threads. Hence w_m is a visible event in G as well as in G'.

  Considering the above mentioned cases $G'.\mathrm{jfe}(w_m, e') \implies w_m \in \mathrm{vis}(G')$ holds and G' satisfies (VISJ) constraint.
- (ICF). We know G satisfies constraint (ICF). Following the construction $e' \in G'.\mathcal{R}$ and following the determinacy condition if $G'. \sim (e_1, e')$ then $e_1 \in \mathsf{Ld}$. Thus $(e_1, e') \in (G'.\mathcal{R} \times G'.\mathcal{R})$ and hence G' satisfies (ICF).
- (ICFJ) From the construction we know there exists no e_1 such that imm(cf)(e_1, e') and $G.\mathrm{rf}(\mathbb{W}^{-1}(wm), e_1)$. Moreover, G satisfies constraint (ICFJ). As a result, G' satisfies (ICFJ).
- (COH) We know that G satisfies (COH) constraint and hence $(G.\mathrm{hb}; G.\mathrm{eco}^?_{\mathrm{strong}})$ is acyclic.
We check if $(G'.\mathrm{hb}; G'.\mathrm{eco}^?_{\mathrm{strong}})$ is acyclic.
The incoming edges to event e' are G'.hb, G'.rf and there is outgoing $G'\mathrm{fr}_{\mathrm{strong}}$ edges.

If $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ forms a cycle then
(i) event e' is in the cycle.
(ii) $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ is in the cycle where w' is some write on x.
(iii) Either $G'.\mathsf{rf}(-, e')$ or $G'.\mathsf{hb}(-, e')$
incoming edge is part of the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.

Analyzing the cases on incoming edges to event e' the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle can be as follows.

– **case** $G'.\mathsf{rf}(-, e')$ completes the the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.
  The $G'.\mathsf{rf}(-, e')$ is either $G'.\mathsf{jf}(w_m, e')$ or there exists w_1 such that $G'.\mathsf{ew}(w_m, w_1)$ and $(w_1, e') \in (G'.\mathsf{ew}; G'.\mathsf{jf})$.
  Thus the cycle can be one of the followings ways.
  (1) $G'.\mathsf{rf}(w_m, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$.
  (2) $G'.\mathsf{rf}(w_1, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_1) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$.
  Also note that $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ implies
  either $G.\mathsf{mo}_{\mathsf{strong}}(w_m, w')$ or $G.\mathsf{mo}_{\mathsf{strong}}(w_1, w')$ already hold in G.
  Considering (1), (2), and possible reasons for $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, we consider following subcases.
  * **subcase**
    (i) $G'.\mathsf{rf}(w_m, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is the cycle, and $G.\mathsf{mo}_{\mathsf{strong}}(w_m, w')$
    (ii) $G'.\mathsf{rf}(w_1, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_1) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is the cycle, and $G.\mathsf{mo}_{\mathsf{strong}}(w_1, w')$
    In case (i) $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ implies
    $(w', w_m) \in (G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ holds in G.
    In that case $(w', w_m) \in (G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ and $G.\mathsf{mo}_{\mathsf{strong}}(w_m, w')$
    form a $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ cycle in G.
    This is not possible as $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic and hence a contradiction.
    Thus $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in this case.
    Following the similar argument $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in case (ii).
  * **subcase**
    (i) $G'.\mathsf{rf}(w_m, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is the cycle, and $G.\mathsf{mo}_{\mathsf{strong}}(w_1, w')$
    (ii) $G'.\mathsf{rf}(w_1, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_1) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is the cycle, and $G.\mathsf{mo}_{\mathsf{strong}}(w_m, w')$
    In case (i) following Lemma 3,
    (a) $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ implies
    $(w', w_m) \in (G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ and in turn $\mathsf{ts}(w') < \mathsf{ts}(w_m)$,
    (b) $G.\mathsf{ew}(w_m, w_1)$ implies $\mathsf{ts}(w_m) = \mathsf{ts}(w_1)$, and
    (c) $G.\mathsf{mo}_{\mathsf{strong}}(w_1, w')$ implies $\mathsf{ts}(w_1) < \mathsf{ts}(w')$.
    The combination of (a), (b), (c) contradicts the total order of timestamps.
    Thus $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in this case.
    Following the similar argument $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in case (ii).

– **case** $G'.\mathsf{hb}(-, e')$ completes the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.
  In this case $G'.\mathsf{rf}(-, e')$ is not part of the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.

Hence $(w', e') \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ and $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$
form the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.
$G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ suggests two possibilities:

* **subcase** $G'.\mathsf{hb}(w_m, w')$.
  Following Lemma 3,
  (a) $\mathsf{ts}(w_m) < \mathsf{ts}(w')$.
  (b) From $(w', e') \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ we know $\mathsf{ts}(w') < \mathsf{ts}(e')$.
  (c) We also know $G'.\mathsf{jf}(w_m, e')$ implies $\mathsf{ts}(w_m) = \mathsf{ts}(e')$.
  (d) However, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ implies $\mathsf{ts}(e') < \mathsf{ts}(w')$.
  The combination of (a), (b), (c), (d) contradicts the total order of timestamps and hence $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in this case.
* **subcase** $G'.\mathsf{hb}(w_1, w')$.
  Following Lemma 3,
  (a) $\mathsf{ts}(w_1) < \mathsf{ts}(w')$.
  (b) From $(w', e') \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ we know $\mathsf{ts}(w') < \mathsf{ts}(e')$.
  (c) We also know $G'.\mathsf{rf}(w_1, e')$ implies $\mathsf{ts}(w_1) = \mathsf{ts}(e')$.
  (d) However, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ implies $\mathsf{ts}(e') < \mathsf{ts}(w')$.
  The combination of (a), (b), (c), (d) contradicts the total order of timestamps and hence $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in this case.

As a result, G' satisfies (COH).

Thus G' is consistent in WEAKEST model.

(2) Condition to show: *The local state of each thread in* MS′ *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*
In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS′ and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo′ we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i)) \cdot e'.\mathsf{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma \cdot e'.\mathsf{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS′ and G'.
Note. This was same as the other scenario when we append a new $\mathsf{St}_o(x, v)$.

(3) Condition to show: *Whenever* $\mathbb{W}'$ *maps an event of* G' *to a message in* MS′, *then the location accessed and the written values match.*
We know $\mathsf{M}' = \mathsf{M}$ and $\mathbb{W}(e') = \perp$. Hence, if $e \neq e'$ then $\mathbb{W}'(e) = \mathbb{W}(e)$. If $e = e'$ then $\mathbb{W}(e') = \perp$ and the assertion holds.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$, *there are corresponding write events in* G' *that are* po*-after* $\mathbb{S}'$.
We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS′, that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

Note. This was same as the other scenario when we append a new $\mathsf{St}_o(x, v)$.

(5) Condition to show: *For every location ℓ and thread j, the thread view of ℓ in the promise state MS' records the timestamp of the maximal write visible to the covered events in G' of thread j.*

Essentially we have to show
$\forall j, \ell.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_j])\}$
For $j \neq i$ or $j = i \wedge \ell \neq x$, it is trivial because $\mathcal{TS}'.V(\ell) = \mathcal{TS}.V(\ell)$.
For $j = i \wedge \ell = x$, we have to show
$\mathcal{TS}'(i).V(x) = \max\{\mathbb{W}'(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_x]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$.
From the definitions we know
(1) $\mathcal{TS}(i).V(x) = \max\{\mathbb{W}(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_x]; G.\mathsf{jf}^?; \mathsf{shb}^?; \mathsf{sc}^?; \mathsf{shb}^?; [\mathbb{S}_i])\}$
(2) $\mathcal{TS}'(i).V(x) = \mathrm{ts}(e') = \mathsf{wm}.ts$.
Following the promising semantics, we know $\mathcal{TS}'(i).V(x)$ extends the thread view of x from $\mathcal{TS}(i).V(x)$ by reading from wm, and $\mathcal{TS}(i).V(x) \leq \mathsf{wm}.ts$.
As a result,
$\mathcal{TS}'(i).V(x) = \mathsf{wm}.ts = \max\{\mathbb{W}'(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_x]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$.
Thus the condition is preserved between MS' and G'.

(6) Condition to show: *The $\mathbb{S}'$ events in G' preserve coherence: $\mathsf{shb}'; \mathsf{seco}'^?$ is irreflexive.*
We know $\mathsf{shb}; \mathsf{seco}^?$ is irreflexive in G.
Let event $a \in \mathbb{S}'$ and assume $(a, e') \in (\mathsf{shb}'; \mathsf{seco}'^?)$ and $(e', a) \in (\mathsf{shb}'; \mathsf{seco}'^?)$.
Following the definitions of shb', seco', and from Lemma 3 (2) we know
$\mathsf{MS}'_a.\mathcal{TS}'(a.\mathrm{tid}).V(x) \leq \mathsf{MS}_{e'}.\mathcal{TS}'(e'.\mathrm{tid}).V(x)$.
However, the only outgoing edge from e' is fr' and from the definition we know $\mathsf{sfr}'(e', b)$ implies that $\mathsf{MS}'_a.\mathcal{TS}'(e'.\mathrm{tid}).V(x) \leq \mathsf{MS}_a.\mathcal{TS}'(e'.\mathrm{tid}).V(x)$.
Hence a contradiction and $\mathsf{shb}'; \mathsf{seco}'^?$ is irreflexive.

(7) Condition to show: *The atomicity condition for update operations holds for $\mathbb{S}'$ events in G'.*

We know that $[G'.\mathsf{U} \cap \mathbb{S}'] = [G.\mathsf{U} \cap \mathbb{S}]$ and $[G.\mathsf{U} \cap \mathbb{S}]; (\mathsf{sfr}; \mathsf{smo}) = \emptyset$ holds.
The e' does not introduce any $[G.\mathsf{U}]; G'.\mathsf{sfr}'$ or $[G.\mathsf{U}]; G'.\mathsf{smo}'$ edge.
As a result, $[G'.\mathsf{U} \cap \mathbb{S}']; (\mathsf{sfr}'; \mathsf{smo}') = \emptyset$ holds.

(8) Condition to show: *The SC fences in G' are appropriately ordered by sc'.*

We know $[G.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}$ holds in G.
From definitions we know, $G'.\mathsf{F}_{\mathsf{sc}} = G.\mathsf{F}_{\mathsf{sc}}$, $\mathsf{sc}' = \mathsf{sc}$, $\mathsf{shb} \subseteq \mathsf{shb}'$, $\mathsf{seco} \subseteq \mathsf{seco}'$.
Consider a, b are two SC fences such that
$(a, b) \in [G.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathsf{F}_{\mathsf{sc}}]$, and $\mathsf{sc}(a, b)$ holds.
In that case $(a, b) \in (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$ holds and $\mathsf{sc}'(a, b)$ holds.
To show $[G'.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$,
we have to show $(b, a) \notin (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
We show that by contradiction. Assume $(b, a) \in (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
This is possible due to the relations created to/from event e'.
Considering the relations in shb' and seco', the incoming relations to event e' are shb' and srf', and the outgoing edges are sfr'.
Thus a $\mathsf{sfr}'(e', w)$ edge where w is a write event occurs in the $(\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$ path from b to a.
In this case the path from b to a is $(b, e') \in \mathsf{shb}'; \mathsf{srf}'^?$ and $(e', a) \in \mathsf{sfr}'; \mathsf{seco}'^?; \mathsf{shb}'$.
It implies $(b, e') \in \mathsf{shb}; \mathsf{srf}'^?$ and $(e', a) \in \mathsf{sfr}'; \mathsf{seco}^?; \mathsf{shb}$.
In this case there exists $w, w' \in G'.\mathcal{W}_x \cap \mathbb{S}$ such that $\mathsf{srf}'(w, e')$ and $\mathsf{sfr}'(e', w')$ holds.

However, from the definitions, in this case $\mathsf{smo}(w, w')$ already holds
and hence $(b, a) \in (\mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb})$ already holds.
This is a contradiction and hence $[G'.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$ holds.

(9) Condition to show: *The behavior of* MS' *matches that of the* $\mathbb{S}'$ *events in* G'.

Essentially we have to show, $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$.
We know $\mathsf{Behavior}(\mathsf{MS}) = \mathsf{Behavior}(G, \mathbb{W}, \mathbb{S})$ holds.
From the definition we know,
$\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(\mathsf{MS})$ and $\mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}') = \mathsf{Behavior}(G, \mathbb{W}, \mathbb{S})$ hold.
As a result, $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$ holds.

Subcase $\exists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \land e'.\mathsf{lab} = \mathsf{Ld}_o(x, v) \land G.\mathsf{jf}(w_m, e')$ **where** $\mathsf{wm} = \mathbb{W}(w_m)$**:**

We take $G' = G$ and let $\mathbb{W}' = \mathbb{W}$.
Based on $\mathbb{W}'$, we derive following definitions in MS'.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathsf{mo}' \triangleq \mathsf{mo}$
- $\mathsf{sc}' \triangleq \mathsf{sc}$
- $\mathsf{spo}' \triangleq (\mathsf{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathsf{srf}' \triangleq \mathsf{srf} \uplus \{(w, e') \mid G'.\mathsf{rf}(w, e') \land w \in \mathbb{S}\}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: G' *is consistent in* WEAKEST *model.*

We know $G'.\mathsf{E} = G.\mathsf{E}$, $G'.\mathsf{po} = G.\mathsf{po}$, $G'.\mathsf{jf} = G.\mathsf{jf}$, and G is consistent. Hence G' is also consistent.

(2) Condition to show: *The local state of each thread in* MS' *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*

In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS' and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo' we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i)) \cdot e'.\mathsf{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma \cdot e'.\mathsf{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS' and G'.

Note. This was same as the other scenario when we append a new $\mathsf{St}_o(x, v)$ or $\mathsf{Ld}_o(x, v)$.

(3) Condition to show: *Whenever* $\mathbb{W}'$ *maps an event of* G' *to a message in* MS'*, then the location accessed and the written values match.*

We know $\mathsf{M}' = \mathsf{M}$ and $\mathbb{W}(e') = \perp$. Hence, if $e \neq e'$ then $\mathbb{W}'(e) = \mathbb{W}(e)$. If $e = e'$ then $\mathbb{W}(e') = \perp$ and the assertion holds.
Note. This was same as the the scenario when we append a new $\mathsf{Ld}_o(x, v)$.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$*, there are corresponding write events in* G' *that are* po*-after* $\mathbb{S}'$*.*

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS′, that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

Note. This was same as the other scenario when we append a new $\mathsf{St}_o(x, v)$ or $\mathsf{Ld}_o(x, v)$.

(5) Condition to show: *For every location* ℓ *and thread* j*, the thread view of* ℓ *in the promise state* MS′ *records the timestamp of the maximal write visible to the covered events in* G' *of thread* j*.*

The argument is analogous to the case when we append a new $\mathsf{Ld}_o(x, v)$.

(6) Condition to show: *The* $\mathbb{S}'$ *events in* G' *preserve coherence:* $\mathsf{shb}'; \mathsf{seco}'^?$ *is irreflexive.*
The argument is analogous to the case when we append a new $\mathsf{Ld}_o(x, v)$.

(7) Condition to show: *The atomicity condition for update operations holds for* $\mathbb{S}'$ *events in* G'*.*

The argument is analogous to the case when we append a new $\mathsf{Ld}_o(x, v)$.

(8) Condition to show: *The* SC *fences in* G' *are appropriately ordered by* sc'*.*

The argument is analogous to the case when we append a new $\mathsf{Ld}_o(x, v)$.

(9) Condition to show: *The behavior of* MS′ *matches that of the* $\mathbb{S}'$ *events in* G'*.*

Essentially we have to show, $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$.
We know $\mathsf{Behavior}(\mathsf{MS}) = \mathsf{Behavior}(G, \mathbb{W}, \mathbb{S})$ holds.
We have $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(\mathsf{MS})$ and $\mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}') = \mathsf{Behavior}(G, \mathbb{W}, \mathbb{S})$ by definition. As a result, $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$ holds.

Case Update $\mathsf{U}(o, x, v, v')$ **reading from message** $\mathsf{wm} = \langle x : v@(-, t], R\rangle$ **and creating message** $m' = \langle x : v'@[-, t'], R'\rangle$**:**

In the event structure we extend the event structure G to G'. We extend the cover set $\mathbb{S}_i$ as well as the relations (spo, srf, smo) to $\mathbb{S}'_i$ along with the respective relations (spo′, srf′, smo′) by including an event e' where

(1) $\mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
(2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
(3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e'.\mathsf{lab}) \in \mathbb{P}(i)$.

In this case the promise machine is updated as follows.
$\mathsf{M}' = \mathsf{M} \uplus \{m'\}$, $\mathcal{S}' = \mathcal{S}$, and $\mathcal{TS}' = \mathcal{TS}[i \mapsto \langle\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle, V', \mathcal{TS}(i).\mathsf{P}\rangle]$ where $V' = \mathcal{TS}(i).V[x \mapsto m'.ts]$.

Now we do a case analysis on whether such an update event e' exists in G or we append a new event.

Subcase $\nexists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\mathsf{lab} = \mathsf{U}(o, x, v, v') \wedge G.\mathsf{rf}(w_m, e')$ **where** $\mathbb{W}(w_m) = \mathsf{wm}$**:**

We create e' such that $e'.\mathsf{lab} = \mathsf{U}_o(x, v, v')$ and append e' to event structure G to create G'. In that case

- $G'.\mathsf{E} = G.\mathsf{E} \uplus \{e'\}$
- $G'.\mathsf{po} = (G.\mathsf{po} \cup \{(e, e') \mid e \in (\mathbb{S}_i \cup \mathbb{S}_0)\})^+$
  $G'.\mathsf{jf} = G.\mathsf{jf} \uplus \{(w_m, e') \mid \mathbb{W}(w_m) = \mathsf{wm} \wedge [\mathbb{S}_0 \cup \mathbb{S}'_i]; G'.\mathsf{po}^?; [\{w_m\}]\}$
- $G'.\mathsf{ew} = G.\mathsf{ew}$

Let: $\mathbb{W}' \triangleq \mathbb{W}[e' \mapsto m']$, and Based on $\mathbb{W}'$, we derive following definitions in MS′.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathsf{mo}' \triangleq \mathsf{mo} \uplus \{(a, e') \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(a).ts < \mathbb{W}'(e').ts\}$
  $\uplus \{(e', a) \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(e').ts < \mathbb{W}'(a).ts\}$
- $\mathsf{sc}' \triangleq \mathsf{sc}$
- $\mathsf{spo}' \triangleq (\mathsf{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathsf{srf}' \triangleq \mathsf{srf} \uplus \{(w, e') \mid G'.\mathsf{rf}(w, e') \wedge w \in \mathbb{S}\}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: *G' is consistent in* WEAKEST *model.*

- (CF) and (CFJ) constraints are preserved in G'. The arguments are analogous to the scenario when we append a new $\mathsf{Ld}_o(x, v)$.
- (VISJ) We study the possible cases of w_m.
  - If $G'.\mathsf{po}(w_m, e')$ then the condition holds as $(w_m, e') \notin G'.\mathsf{rfe}$.
  - We will show that G' satisfies (CFJ) constraint. Hence w_m cannot be in conflict with e', that is, $(w_m, e') \notin G'.\mathsf{cf}$.
  - w_m is in different thread and $G'.\mathsf{jfe}(w_m, e')$ holds. We know that $G \sim_{\{i\}}$ MS and the simulation rules ensures that there is no *invisible* event in the $(\mathsf{T} \setminus \{i\})$ threads. Hence w_m is a visible event in G as well as in G'.

  Considering the above mentioned cases $G'.\mathsf{jfe}(w_m, e') \implies w_m \in \mathsf{vis}(G')$ holds and G' satisfies (VISJ) constraint.

  Note. This was same as the other scenario when we append a new $\mathsf{Ld}_o(x, v)$.
- (ICF). We know G satisfies constraint (ICF). Following the construction $e' \in G'.\mathcal{R}$ and following the determinacy condition if $G'. \sim (e_1, e')$ then $e_1 \in \mathsf{Ld}$ or $e_1 \in \mathsf{U}$. Thus $(e_1, e') \in (G'.\mathcal{R} \times G'.\mathcal{R})$ and hence G' satisfies (ICF).

  Note. This was same as the other scenario when we append a new $\mathsf{Ld}_o(x, v)$.
- (ICFJ) From the construction we know there exists no e_1 such that $\mathsf{imm}(\mathsf{cf})(e_1, e')$ and $G.\mathsf{rf}(\mathbb{W}^{-1}(wm), e_1)$. Moreover, G satisfies constraint (ICFJ). As a result, G' satisfies (ICFJ).
- (COH) We know that G satisfies (COH) constraint and hence $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic.

  We check if $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic.

  The incoming edges to event e' are $G'.\mathsf{hb}$, $G'.\mathsf{jf}$ and there is outgoing $G'\mathsf{fr}_{\mathsf{strong}}$ edges.

  If $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ forms a cycle then

  (i) event e' is in the cycle.

  (ii) $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ is in the cycle where w' is some write on x.

  (iii) Either $G'.\mathsf{rf}(-, e')$ or $G'.\mathsf{hb}(-, e')$ incoming edge is part of the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.

  Analyzing the cases on incoming edges to event e' the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle can be as follows.
  - **case** $G'.\mathsf{rf}(-, e')$ completes the the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.

    The $G'.\mathsf{rf}(-, e')$ is either $G'.\mathsf{jf}(w_m, e')$ or there exists w_1 such that $G'.\mathsf{ew}(w_m, w_1)$ and $(w_1, e') \in (G'.\mathsf{ew}; G'.\mathsf{jf})$.

    Thus the cycle can be one of the followings ways.

    (1) $G'.\mathsf{rf}(w_m, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$.

    (2) $G'.\mathsf{rf}(w_1, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_1) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$.

    Also note that $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ implies either $G.\mathsf{mo}_{\mathsf{strong}}(w_m, w')$ or $G.\mathsf{mo}_{\mathsf{strong}}(w_1, w')$ already hold in G.

    Considering (1), (2), and possible reasons for $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, we consider following subcases.

  * **subcase**
    (i) $G'.\mathsf{rf}(w_m, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is the cycle, and $G.\mathsf{mo}_{\mathsf{strong}}(w_m, w')$
    (ii) $G'.\mathsf{rf}(w_1, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_1) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is the cycle, and $G.\mathsf{mo}_{\mathsf{strong}}(w_1, w')$
    In case (i) $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ implies $(w', w_m) \in (G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ holds in G.
    In that case $(w', w_m) \in (G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ and $G.\mathsf{mo}_{\mathsf{strong}}(w_m, w')$ forms a $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ cycle in G.
    This is not possible as $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic and hence a contradiction.
    Thus $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in this case.
    Following the similar argument $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in case (ii).
  * **subcase**
    (i) $G'.\mathsf{rf}(w_m, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is the cycle, and $G.\mathsf{mo}_{\mathsf{strong}}(w_1, w')$
    (ii) $G'.\mathsf{rf}(w_1, e')$, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$, and $(w', w_1) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is the cycle, and $G.\mathsf{mo}_{\mathsf{strong}}(w_m, w')$
    In case (i) following Lemma 3,
    (a) $(w', w_m) \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ implies $(w', w_m) \in (G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ and hence $\mathsf{ts}(w') < \mathsf{ts}(w_m)$,
    (b) $G.\mathsf{ew}(w_m, w_1)$ implies $\mathsf{ts}(w_m) = \mathsf{ts}(w_1)$, and
    (c) $G.\mathsf{mo}_{\mathsf{strong}}(w_1, w')$ implies $\mathsf{ts}(w_1) < \mathsf{ts}(w')$.
    The combination of (a), (b), (c) contradicts the total order of timestamps.
    Thus $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in this case.
    Following the similar argument $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in case (ii).

- **case** $G'.\mathsf{hb}(-, e')$ completes the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.
  In this case $G'.\mathsf{rf}(-, e')$ is not part of the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.
  Hence $(w', e') \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ and $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ forms the $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.
  $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ suggests two possibilities:
  * **subcase** $G'.\mathsf{hb}(w_m, w')$.
    Following Lemma 3,
    (a) $\mathsf{ts}(w_m) < \mathsf{ts}(w')$.
    (b) From $(w', e') \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ we know $\mathsf{ts}(w') < \mathsf{ts}(e')$.
    (c) We also know $G'.\mathsf{jf}(w_m, e')$ implies $\mathsf{ts}(w_m) < \mathsf{ts}(e')$.
    (d) However, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ implies $\mathsf{ts}(e') < \mathsf{ts}(w')$.
    The combination of (a), (b), (c), (d) contradicts the total order of timestamps and hence $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in this case.
  * **subcase** $G'.\mathsf{hb}(w_1, w')$.
    Following Lemma 3,
    (a) $\mathsf{ts}(w_1) < \mathsf{ts}(w')$.
    (b) From $(w', e') \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ we know $\mathsf{ts}(w') < \mathsf{ts}(e')$.
    (c) We also know $G'.\mathsf{rf}(w_1, e')$ implies $\mathsf{ts}(w_1) = \mathsf{ts}(e')$.
    (d) However, $G'.\mathsf{fr}_{\mathsf{strong}}(e', w')$ implies $\mathsf{ts}(e') < \mathsf{ts}(w')$.

The combination of (a), (b), (c), (d) contradicts the total order of timestamps and hence $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic in this case.

As a result, G' satisfies (COH).

Thus G' is consistent in WEAKEST model.

(2) Condition to show: *The local state of each thread in* MS' *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*

In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS' and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo' we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\cdot e'.\mathsf{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma\cdot e'.\mathsf{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS' and G'.

Note. This was similar to the other scenario when we append a new $\mathsf{St}_o(x, v)$.

(3) Condition to show: *Whenever* $\mathbb{W}'$ *maps an event of* G' *to a message in* MS'*, then the location accessed and the written values match.*

We know that the event to message mappings for existing events in $G.\mathsf{E}$ and messages M do not change.

$$\forall e \in G'.\mathsf{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$$

If $e = e'$ then $\mathbb{W}'(e') = m'$ and $e'.\mathsf{loc} = m'.\mathsf{loc} = x$ and $e'.\mathsf{wval} = m'.\mathsf{wval} = v$.
Hence $\mathbb{W}'$ preserves the condition.
Note. This was similar to the other scenario when we append a new $\mathsf{St}_o(x, v)$.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$*, there are corresponding write events in* G' *that are* po*-after* $\mathbb{S}'$*.*

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS', that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.
Note. This was similar to the other scenario when we append a new $\mathsf{St}_o(x, v)$.

(5) Condition to show: *For every location* ℓ *and thread* j*, the thread view of* ℓ *in the promise state* MS' *records the timestamp of the maximal write visible to the covered events in* G' *of thread* j*.*

Essentially we have to show
$\forall j, \ell.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_j])\}$
For $j \neq i$ or $j = i \wedge \ell \neq x$, it is trivial because $\mathcal{TS}'.V(\ell) = \mathcal{TS}.V(\ell)$.
For $j = i \wedge \ell = x$, from the definition we know
$\mathcal{TS}(i).V(x) = \max\{\mathbb{W}(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G.\mathsf{jf}^?; \mathsf{shb}^?; \mathsf{sc}^?; \mathsf{shb}^?; [\mathbb{S}_i])\}$
Following the promising semantics, we know $\mathcal{TS}'(i).V(x)$ extends the thread view of x from $\mathcal{TS}(i).V(x)$ by reading from wm, and hence $\mathcal{TS}(i).V(x) < \mathsf{wm}.ts$.
Moreover, following the semantics of update operation in promise machine $\mathsf{wm}.ts < m'.ts$.
Hence following the construction,

$\mathcal{TS}'(i).V(x) = m'.ts = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$.
Thus the condition is preserved between MS' and G'.

(6) Condition to show: *The $\mathbb{S}'$ events in G' preserve coherence:* $\mathsf{shb}'; \mathsf{seco}'^?$ *is irreflexive.*
The argument is analogous to the case when we append a new $\mathsf{St}_o(x, v)$.

(7) Condition to show: *The atomicity condition for update operations holds for $\mathbb{S}'$ events in G'.*
Assume $[G'.\mathsf{U} \cap \mathbb{S}']; (\mathsf{sfr}'; \mathsf{smo}') \neq \emptyset$.
We know that $[G.\mathsf{U} \cap \mathbb{S}]; (\mathsf{sfr}; \mathsf{smo}) = \emptyset$ holds.
Hence e' is involved in atomicity violation. In that case two possibilities as follows:

- **case** There exists an update $u \in (G.\mathsf{U}_x \cap \mathbb{S})$ such that $\mathsf{sfr}(u, e')$ and $\mathsf{smo}(e', u)$ holds.
  Assume u reads from w_1, that is, $\mathsf{srf}(w_1, u)$.
  $\mathsf{sfr}'(u, e')$ implies that $\mathsf{mo}(w_1, e')$ holds.
  $\mathsf{mo}'(w_1, e')$ implies $\mathbb{W}'(w_1).ts < \mathbb{W}'(e').ts$.
  However, $\mathsf{srf}'(w_1, u)$ implies $\mathbb{W}'(w_1).ts < \mathbb{W}'(u).ts$
  and there is no write on x in the time range $(\mathbb{W}'(w_1).ts, \mathbb{W}'(u).ts]$, that is,
  $\nexists w' \in \mathbb{S}' \cap G'.\mathcal{W}_x.\ \mathbb{W}'(w_1).ts < \mathbb{W}'(w').ts < \mathbb{W}'(u).ts$.
  As a result, $\mathbb{W}'(w_1).ts < \mathbb{W}'(e').ts < \mathbb{W}'(u).ts$ is not possible and
  hence $\mathbb{W}'(u).ts < \mathbb{W}'(e').ts$ which implies $\mathsf{smo}'(u, e')$.
  $\mathsf{smo}'(u, e')$ and $\mathsf{smo}'(e', u)$ both cannot hold.
  Hence a contradiction and in this case atomicity holds in $\mathbb{S}'$ events in G'.
- **case** There exists a write $w' \in (G'.\mathcal{W}_x \cap \mathbb{S}')$ such that $\mathsf{sfr}'(e', w')$ and $\mathsf{smo}'(w', e')$ hold.
  $\mathsf{sfr}'(e', w')$ implies $\mathsf{smo}'(w, w')$, that is, $\mathbb{W}'(w).ts < \mathbb{W}'(w').ts$.
  However, $\mathsf{srf}'(w, e')$ implies $\mathbb{W}'(w).ts < \mathbb{W}'(e').ts$
  and there is no write on x in the time range $(\mathbb{W}'(w).ts, \mathbb{W}'(e').ts]$, that is,
  $\nexists w' \in (G'.\mathcal{W}_x \cap \mathbb{S}').\ \mathbb{W}'(w).ts < \mathbb{W}'(w').ts < \mathbb{W}'(e').ts$.
  As a result, neither $\mathbb{W}'(w).ts < \mathbb{W}'(e').ts < \mathbb{W}'(e').ts$ is not possible and
  hence $\mathbb{W}'(e').ts < \mathbb{W}'(w').ts$ which implies $\mathsf{smo}'(e', w')$.
  $\mathsf{smo}'(e', w')$ and $\mathsf{smo}'(w', e')$ both cannot hold.
  Hence a contradiction and in this case atomicity holds in $\mathbb{S}'$ events in G'.

(8) Condition to show: *The SC fences in G' are appropriately ordered by* sc'.
We know $[G.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}$ holds in G.
From definitions we know, $G'.\mathsf{F}_{\mathsf{sc}} = G.\mathsf{F}_{\mathsf{sc}}$, $\mathsf{sc}' = \mathsf{sc}$, $\mathsf{shb} \subseteq \mathsf{shb}'$, $\mathsf{seco} \subseteq \mathsf{seco}'$.
Consider a, b are two SC fences such that $(a, b) \in [G.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathsf{F}_{\mathsf{sc}}]$, and $\mathsf{sc}(a, b)$ holds.
In that case $(a, b) \in (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$ holds and $\mathsf{sc}'(a, b)$ holds.
To show $[G'.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$,
we have to show $(b, a) \notin (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
We show that by contradiction. Assume $(b, a) \in (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
This is possible due to the relations created to/from event e'.
Considering the relations in shb' and seco', the incoming relations to event e' are shb', srf', sfr', smo' and the outgoing edges are sfr', smo'.
Since e' is an update, for a write event w_1, relation $\mathsf{sfr}'(u, w_1)$ implies $\mathsf{smo}'(u, w_1)$.
Hence we consider only smo' as outgoing edge.
In this case the path from b to a is $(b, e') \in \mathsf{shb}'; \mathsf{seco}'^?$ and $(e', a) \in \mathsf{smo}'; \mathsf{seco}'^?; \mathsf{shb}'$.
As there is no outgoing srf edge from e', no new synchronization edge is created, that is, $\mathsf{ssw}' = \mathsf{ssw}$.
We analyze the cases of $(b, e') \in \mathsf{shb}'; \mathsf{seco}'^?$.

In this case there exists some event c such that

- $\mathsf{shb}'(b, e')$.
  Two possible subcases:
  - **subcase** In this case $\mathsf{shb}(b, e)$ and $\mathsf{spo}'(e, e')$ holds.
    So $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) \leq \mathsf{MS}_e.\mathcal{TS}(e.\mathsf{tid}).V(x) < \mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x)$.
  - **subcase** $\mathsf{shb}(b, c)$ and $\mathsf{ssw}'(c, e')$ holds.
    Hence $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) \leq \mathsf{MS}_c.\mathcal{TS}(c.\mathsf{tid}).V(x)$ holds.
    Moreover, consider the cases of ssw', following from Lemma 3, we can show that $\mathsf{MS}_c.\mathcal{TS}(c.\mathsf{tid}).V(x) < \mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x)$ holds.

  Considering both subcases $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) < \mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x)$ holds.
- $\mathsf{shb}'; \mathsf{seco}'(b, c)$ and $\mathsf{srf}'(c, e')$.
  Hence $\mathsf{shb}; \mathsf{seco}(b, c)$ and $\mathsf{srf}'(c, e')$ holds.
  As a result, following promising semantics,
  $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) \leq \mathsf{MS}_c.\mathcal{TS}(c.\mathsf{tid}).V(x) < \mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x)$.
- $\mathsf{shb}'; \mathsf{seco}'(b, c)$ and $\mathsf{smo}'(c, e')$.
  Hence $\mathsf{shb}; \mathsf{seco}(b, c)$ and $\mathsf{smo}'(c, e')$ holds.
  As a result, following promising semantics,
  $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) \leq \mathsf{MS}_c.\mathcal{TS}(c.\mathsf{tid}).V(x) < \mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x)$.
- $\mathsf{shb}'; \mathsf{seco}'(b, c)$ and $\mathsf{sfr}'(c, e')$.
  Hence $\mathsf{shb}; \mathsf{seco}(b, c)$ and $\mathsf{sfr}'(c, e')$ holds.
  As a result, following promising semantics,
  $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) \leq \mathsf{MS}_c.\mathcal{TS}(c.\mathsf{tid}).V(x) < \mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x)$.

Now we analyze $(e', a) \in \mathsf{smo}'; \mathsf{seco}'^?; \mathsf{shb}'$.
In this case there exist a write $w \in \mathbb{S}$ such that
$\mathsf{smo}'(e', w)$ and $(w, a) \in \mathsf{seco}^?; \mathsf{shb}$ holds.
Hence $\mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x) < \mathsf{MS}_w.\mathcal{TS}(w.\mathsf{tid}).V(x) \leq \mathsf{MS}_a.\mathcal{TS}(a.\mathsf{tid}).V(x)$.
As a result, in all cases $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) < \mathsf{MS}_a.\mathcal{TS}(a.\mathsf{tid}).V(x)$ holds.
However, we know that $\mathsf{sc}(a, b)$ holds and hence $\mathsf{MS}_a.V \leq \mathsf{MS}_b.V$.
This is a contradiction and hence $(b, a) \notin (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
As a result, $[G'.\mathsf{F_{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathsf{F_{sc}}] \subseteq \mathsf{sc}'$ holds.

(9) Condition to show: *The behavior of* MS' *matches that of the* $\mathbb{S}'$ *events in* G'*.*
The argument is analogous to the case when we append a new $\mathsf{St}_o(x, v)$.

Subcase $\exists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\mathsf{lab} = \mathsf{U}(o, x, v, v') \wedge G.\mathsf{jf}(w_m, e')$ **where** $\mathsf{wm} = \mathbb{W}(w_m)$**:**

We take $G' = G$ and let $\mathbb{W}' = \mathbb{W}[e' \mapsto m']$.
Based on $\mathbb{W}'$, we derive following definitions in MS'.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathsf{mo}' \triangleq \mathsf{mo} \uplus \{(a, e') \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(a).ts < \mathbb{W}'(e').ts\}$
  $\uplus \{(e', a) \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(e').ts < \mathbb{W}'(a).ts\}$
- $\mathsf{sc}' \triangleq \mathsf{sc}$
- $\mathsf{spo}' \triangleq (\mathsf{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathsf{srf}' \triangleq \mathsf{srf} \uplus \{(w, e') \mid G'.\mathsf{rf}(w, e') \wedge w \in \mathbb{S}\}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: G' *is consistent in* WEAKEST *model.*

We know $G'.\mathsf{E} = G.\mathsf{E}$, $G'.\mathsf{po} = G.\mathsf{po}$, $G'.\mathsf{jf} = G.\mathsf{jf}$, and G is consistent. Hence G' is also consistent in WEAKEST model.

(2) Condition to show: *The local state of each thread in* MS' *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*

In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS′ and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo' we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\cdot e'.\mathsf{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma\cdot e'.\mathsf{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS′ and G'.

Note. This was same as the other scenario when we append a new $\mathsf{St}_o(x, v)$.

(3) Condition to show: *Whenever* $\mathbb{W}'$ *maps an event of* G' *to a message in* MS'*, then the location accessed and the written values match.*

The event to message mappings for existing events in $G.\mathsf{E}$ and messages M do not change.

$$\forall e \in G'.\mathsf{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$$

If $e = e'$ then $\mathbb{W}'(e') = \mathsf{wmsg}(\mathsf{op}) = m'$ and $e'.\mathsf{loc} = m'.\mathsf{loc} = x$ and $e.\mathsf{wval} = m'.\mathsf{wval} = v$.
Hence $\mathbb{W}'$ preserves the condition.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$*, there are corresponding write events in* G' *that are* po*-after* $\mathbb{S}'$*.*

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS′, that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

Note. This was same as the other scenario when we append a new $\mathsf{St}_o(x, v)$.

(5) Condition to show: *For every location* ℓ *and thread* j*, the thread view of* ℓ *in the promise state* MS' *records the timestamp of the maximal write visible to the covered events in* G' *of thread* j*.*

The argument is analogous to the case when we append a new $\mathsf{U}_o(x, v, v')$.

(6) Condition to show: *The* $\mathbb{S}'$ *events in* G' *preserve coherence:* $\mathsf{shb}';\mathsf{seco}'^{?}$ *is irreflexive.*
The argument is analogous to the case when we append a new $\mathsf{U}_o(x, v, v')$.

(7) Condition to show: *The atomicity condition for update operations hold for* $\mathbb{S}'$ *events in* G'*.*
The argument is analogous to the case when we append a new $\mathsf{U}_o(x, v, v')$.

(8) Condition to show: *The* SC *fences in* G' *are appropriately ordered by* sc'*.*
We know $[G.\mathsf{F}_{\mathsf{sc}}];\mathsf{shb} \cup \mathsf{shb};\mathsf{seco};\mathsf{shb};[G.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}$ holds in G.

The argument is analogous to the case when we append a new $\mathsf{U}_o(x, v, v')$.

(9) Condition to show: *The behavior of* MS' *matches that of the* $\mathbb{S}'$ *events in* G'*.*

The argument is analogous to the case when we append a new $\mathsf{U}_o(x, v, v')$.

Case Release fence F_{REL}:

In the event structure we extend the event structure G to G'. We extend the cover set $\mathbb{S}_i$ as well as the relations (spo, srf, smo) to $\mathbb{S}'_i$ along with the respective relations (spo′, srf′, smo′) by including an event e' where

(1) $\mathrm{dom}(G.\mathrm{po};[\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
(2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
(3) $\mathit{labels}(\mathrm{sequence}_{G.\mathrm{po}}(\mathbb{S}_i)).(e'.\mathrm{lab}) \in \mathbb{P}(i)$.

In this case the promise machine is updated as follows.
$\mathsf{M}' = \mathsf{M}, \mathcal{S}' = \mathcal{S}$,
and $\mathcal{TS}' = \mathcal{TS}[i \mapsto \langle\langle \mathbb{P}(i), \mathit{labels}(\mathrm{sequence}_{\mathrm{spo}'}(\mathbb{S}'_i))\rangle, \langle V.\mathrm{cur}, V.\mathrm{acq}, V.\mathrm{rel}'\rangle, \mathcal{TS}(i).\mathsf{P}\rangle]$

Now we do a case analysis on whether such an release fence event e' exists in G or we append a new event.

Subcase $\nexists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathrm{dom}(G.\mathrm{po};[\{e'\}]) \subseteq \mathbb{S}_i \wedge e'.\mathrm{lab} = \mathsf{F}_{\text{REL}}$:

We create e' such that $e'.\mathrm{lab} = \mathsf{F}_{\text{REL}}$ and append e' to event structure G to create G'. Then,

- $G'.\mathsf{E} = G.\mathsf{E} \uplus \{e' \mid e'.\mathrm{lab} = \mathsf{F}_{\text{REL}}\}$
  $G'.\mathrm{po} = (G.\mathrm{po} \cup \{(e, e') \mid e \in (\mathbb{S}_i \cup \mathbb{S}_0)\})^+$
- $G'.\mathrm{jf} = G.\mathrm{jf}$
- $G'.\mathrm{ew} = G.\mathrm{ew}$

Let: $\mathbb{W}' \triangleq \mathbb{W}$.

Based on $\mathbb{W}'$, we derive following definitions in MS′.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathrm{mo}' \triangleq \mathrm{mo}$
- $\mathrm{sc}' \triangleq \mathrm{sc}$
- $\mathrm{spo}' \triangleq (\mathrm{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathrm{srf}' \triangleq \mathrm{srf}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: *G' is consistent in WEAKEST model.*

- (CF) and (CFJ) constraints are preserved in G'. The arguments are analogous to the scenario when we append a new $\mathrm{St}_o(x, v)$.
- (VISJ) Constraint (VISJ) is preserved in G' as $G'.\mathrm{jf} = G.\mathrm{jf}$ and G satisfies constraint (VISJ).
- (ICF)
  We know that G satisfies (ICF). Suppose there exists an event $e_1 \in G$ which is in immediate conflict with e' in G', that is $G'. \sim (e_1, e')$ holds.
  Then (1) $\mathrm{dom}(G.\mathrm{po};[\{e_1\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e_1 \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathrm{sequence}_{G.\mathrm{po}}(\mathbb{S}_i)).(e_1.\mathrm{lab}) \in \mathbb{P}(i)$.
  However, from definition of e' we already know that
  (1) $\mathrm{dom}(G.\mathrm{po};[\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathrm{sequence}_{G.\mathrm{po}}(\mathbb{S}_i)).(e'.\mathrm{lab}) \in \mathbb{P}(i)$.
  Hence following the determinacy condition we know either $e_1 = e'$ or there exists no such e_1.
  Hence (ICF) is preserved in G'.
  Note. This was similar to the scenario when we append a new $\mathrm{St}_o(x, v)$.
- (ICFJ) Constraint (ICFJ) is preserved in G' as $e' \notin \mathcal{R}$ and G satisfies constraint (ICFJ).

- (COH) We know G preserves (COH) constraint, that is, $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic. The incoming edges to event e' are $G'.\mathsf{po}$ and there is no outgoing edge concerning $G'.\mathsf{hb}$ or $G'.\mathsf{eco}_{\mathsf{strong}}$. As a result, $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic and G' preserves (COH) constraint.

(2) Condition to show: *The local state of each thread in* MS' *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*
In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS' and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo' we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i)) \cdot e'.\mathsf{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma \cdot e'.\mathsf{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS' and G'.

(3) Condition to show: *Whenever* $\mathbb{W}'$ *maps an event of* G' *to a message in* MS'*, then the location accessed and the written values match.*
We know that the event to message mappings for existing events in $G.\mathsf{E}$ and messages M do not change, that is, $\forall e \in G'.\mathsf{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$. If $e = e'$ then $\mathbb{W}'(e') = \perp$.
Hence $\mathbb{W}'$ preserves the condition.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$*, there are corresponding write events in* G' *that are* po*-after* $\mathbb{S}'$.
We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS', that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

(5) Condition to show: *For every location* ℓ *and thread* j*, the thread view of* ℓ *in the promise state* MS' *records the timestamp of the maximal write visible to the covered events in* G' *of thread* j.
Essentially we have to show
$\forall j, \ell.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_j])\}$
We know the relation holds in G.
In G', for all j, ℓ, $\mathcal{TS}'(j).V(\ell) = \mathcal{TS}(j).V(\ell)$ considering the mapping of $\mathcal{TS}'$.
Hence $\mathcal{TS}'$ satisfies the same condition and the relation holds between MS' and G'.

(6) Condition to show: *The* $\mathbb{S}'$ *events in* G' *preserve coherence:* $\mathsf{shb}'; \mathsf{seco}'^?$ *is irreflexive.*
We know $\mathsf{shb}; \mathsf{seco}^?$ is irreflexive.
Following the definition of components of shb' and $\mathsf{seco}'^?$ we know $\mathsf{shb}'; \mathsf{seco}'^?$ is irreflexive.

(7) Condition to show: *The atomicity condition for update operations holds for* $\mathbb{S}'$ *events in* G'.
We know that $[G'.\mathsf{U} \cap \mathbb{S}'] = [G.\mathsf{U} \cap \mathbb{S}]$ and $[G.\mathsf{U} \cap \mathbb{S}]; (\mathsf{sfr}; \mathsf{smo}) = \emptyset$ holds.
The e' does not introduce any $[G.\mathsf{U}]; G'.\mathsf{sfr}'$ or $[G.\mathsf{U}]; G'.\mathsf{smo}'$ edge.
As a result, $[G'.\mathsf{U} \cap \mathbb{S}']; (\mathsf{sfr}'; \mathsf{smo}') = \emptyset$ holds.

(8) Condition to show: *The* SC *fences in* G' *are appropriately ordered by* sc'.
There is no outgoing edge from e' to any event in $\mathbb{S}'$.
Hence event e' cannot introduce a new $(\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$ path between two SC fences.

Hence $[G'.\mathcal{F}_{\text{SC}}]; \text{shb}' \cup \text{shb}'; \text{seco}'; \text{shb}'; [G'.\mathcal{F}_{\text{SC}}]$
implies $[G.\mathcal{F}_{\text{SC}}]; \text{shb} \cup \text{shb}; \text{seco}; \text{shb}; [G.\mathcal{F}_{\text{SC}}]$.
We also know $\text{sc}' = \text{sc}$.
We also know $[G.\mathcal{F}_{\text{SC}}]; \text{shb} \cup \text{shb}; \text{seco}; \text{shb}; [G.\mathcal{F}_{\text{SC}}] \subseteq \text{sc}$.
Hence $[G'.\mathcal{F}_{\text{SC}}]; \text{shb}' \cup \text{shb}'; \text{seco}'; \text{shb}'; [G'.\mathcal{F}_{\text{SC}}] \subseteq \text{sc}'$ holds.

(9) Condition to show: *The behavior of* MS' *matches that of the* $\mathbb{S}'$ *events in* G'.
Essentially we have to show, $\text{Behavior}(\text{MS}') = \text{Behavior}(G', \mathbb{W}', \mathbb{S}')$.
We know $\text{Behavior}(\text{MS}) = \text{Behavior}(G, \mathbb{W}, \mathbb{S})$ holds.
From the definition we know,
$\text{Behavior}(\text{MS}') = \text{Behavior}(\text{MS})$ and $\text{Behavior}(G', \mathbb{W}', \mathbb{S}') = \text{Behavior}(G, \mathbb{W}, \mathbb{S})$ hold.
As a result, $\text{Behavior}(\text{MS}') = \text{Behavior}(G', \mathbb{W}', \mathbb{S}')$ holds.

Subcase $\exists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \text{dom}(G.\text{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\text{lab} = \mathsf{F}_{\text{REL}}$**:**
Note that promising semantics does not promise over a release fence. As a result, the certificate steps do not have any release fence. Hence there is no existing release fence event correspond to any certificate step which can be referred later in the simulation step. As a result, this case is not possible.

Case Acquire fence F_{ACQ}**:**
In the event structure we extend the event structure G to G'. We extend the cover set $\mathbb{S}_i$ as well as the relations (spo, srf, smo) to $\mathbb{S}'_i$ along with the respective relations (spo′, srf′, smo′) by including an event e' where
(1) $\text{dom}(G.\text{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
(2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
(3) $\textit{labels}(\text{sequence}_{G.\text{po}}(\mathbb{S}_i)).(e'.\text{lab}) \in \mathbb{P}(i)$.
In this case the promise machine is updated as follows.
$\mathsf{M}' = \mathsf{M}$, $\mathcal{S}' = \mathcal{S}$, and
$\mathcal{TS}' = \mathcal{TS}[i \mapsto \langle\langle \mathbb{P}(i), \textit{labels}(\text{sequence}_{\text{spo}'}(\mathbb{S}'_i))\rangle, \langle V.\text{cur}', V.\text{acq}, V.\text{rel}\rangle, \mathcal{TS}(i).\mathsf{P}\rangle]$
Now we do a case analysis on whether such an acquire fence event e' exists in G or we append a new event.

Subcase $\nexists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \text{dom}(G.\text{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\text{lab} = \mathsf{F}_{\text{ACQ}}$**:**
We create e' such that $e'.\text{lab} = \mathsf{F}_{\text{ACQ}}$ and append e' to event structure G to create G'. Then,

- $G'.\mathsf{E} = G.\mathsf{E} \uplus \{e' \mid e'.\text{lab} = \mathcal{F}_{\text{ACQ}}\}$ $G'.\text{po} = G.\text{po} \cup \{(e, e') \mid e \in (\mathbb{S}_i \cup \mathbb{S}_0)\}$
- $G'.\text{jf} = G.\text{jf}$
- $G'.\text{ew} = G.\text{ew}$

Let: $\mathbb{W}' \triangleq \mathbb{W}$.
Based on $\mathbb{W}'$, we derive following definitions in MS'.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\text{mo}' \triangleq \text{mo}$
- $\text{sc}' \triangleq \text{sc}$
- $\text{spo}' \triangleq (\text{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\text{srf}' \triangleq \text{srf}$

Note that there may be incoming synchronization edges to the acquire fence, that is, $\text{ssw} \subseteq \text{ssw}'$ and hence $\text{shb} \subseteq \text{shb}'$.
Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: *G' is consistent in* WEAKEST *model.*
- (CF) The constraint is preserved in G'. The argument is analogous to the scenario when we append a new $\mathsf{Ld}_o(x, v)$.
- (CFJ) Constraint (CFJ) is preserved in G'. The argument is analogous to the scenario when we append a new $\mathsf{St}_o(x, v)$.
- (VISJ) Constraint (VISJ) is preserved in G' as $G'.\mathsf{jf} = G.\mathsf{jf}$ and G satisfies constraint (VISJ).
- (ICF)
  We know that G satisfies (ICF). Suppose there exists an event $e_1 \in G$ which is in immediate conflict with e' in G', that is $G'. \sim (e_1, e')$ holds.
  Then (1) $\mathsf{dom}(G.\mathsf{po}; [\{e_1\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e_1 \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e_1.\mathsf{lab}) \in \mathbb{P}(i)$.
  However, from definition of e' we already know that
  (1) $\mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e'.\mathsf{lab}) \in \mathbb{P}(i)$.
  Hence following the determinacy condition we know either $e_1 = e'$ or there exists no such e_1.
  Hence (ICF) is preserved in G'.
  Note. This was similar to the scenario when we append a new $\mathcal{F}_{\text{REL}}$.
- (ICFJ) Constraint (ICFJ) is preserved in G' as $e' \notin \mathcal{R}$ and G satisfies constraint (ICFJ).
- (COH) We know G preserves (COH) constraint, that is, $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic. The incoming edges to event e' are $G'.\mathsf{po}$ and $G'.\mathsf{hb}$ (due to $G'.\mathsf{sw}$ edges), and there is no outgoing edge concerning $G'.\mathsf{hb}$ or $G'.\mathsf{eco}_{\mathsf{strong}}$. As a result, $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic and G' preserves (COH) constraint.

(2) Condition to show: *The local state of each thread in* MS' *contains the program of that thread along with the sequence of covered events in G' of that thread.*

In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS' and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo' we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i)) \cdot e'.\mathsf{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma \cdot e'.\mathsf{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS' and G'.

(3) Condition to show: *Whenever $\mathbb{W}'$ maps an event of G' to a message in* MS'*, then the location accessed and the written values match.*

We know that the event to message mappings for existing events in $G.\mathsf{E}$ and messages M do not change, that is, $\forall e \in G'.\mathsf{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$. If $e = e'$ then $\mathbb{W}'(e') = \perp$.
Hence $\mathbb{W}'$ preserves the condition.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$*, there are corresponding write events in G' that are* po*-after* $\mathbb{S}'$.

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS$'$, that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

(5) Condition to show: *For every location ℓ and thread j, the thread view of ℓ in the promise state MS$'$ records the timestamp of the maximal write visible to the covered events in G' of thread j.*
Essentially we have to show
$\forall j, \ell.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_j])\}$.
We know the relation holds in G.
In G',
- for all $j \neq i$, $\mathcal{TS}'(j).V(\ell) = \mathcal{TS}(j).V(\ell)$ considering the mapping of $\mathcal{TS}'$.
- For $j = i$, $\mathcal{TS}'(j).V.\mathrm{cur} = \mathcal{TS}(j).V.\mathrm{acq}$.
  We know that $\mathcal{TS}(i).V.\mathrm{cur} \leq \mathcal{TS}(i).V.\mathrm{acq}$ for all location ℓ.
  As a result, in this case $\mathcal{TS}'(i).V.\mathrm{cur} \geq \mathcal{TS}(i).V.\mathrm{cur}$.
  Hence
  $\forall \ell.\ \mathcal{TS}'(i).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$ holds.

Thus the relation holds between MS$'$ and G'.

(6) Condition to show: *The $\mathbb{S}'$ events in G' preserve coherence: $\mathsf{shb}'; \mathsf{seco}'^?$ is irreflexive.*
We know $\mathsf{shb}; \mathsf{seco}^?$ is irreflexive.
Following the definition of components of shb' and $\mathsf{seco}'^?$ we know $\mathsf{shb}'; \mathsf{seco}'^?$ is irreflexive.

(7) Condition to show: *The atomicity condition for update operations holds for $\mathbb{S}'$ events in G'.*
The argument is analogous to the case when we append a new F_{REL}.

(8) Condition to show: *The* SC *fences in G' are appropriately ordered by sc'.*
The argument is analogous to the case when we append a new F_{REL}.

(9) Condition to show: *The behavior of* MS$'$ *matches that of the $\mathbb{S}'$ events in G'.*
The argument is analogous to the case when we append a new F_{REL}.

Subcase $\exists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathrm{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\mathrm{lab} = \mathsf{F}_{\mathsf{ACQ}}$:
Note that promising semantics does not promise over an acquire fence. As a result, the certificate steps do not have any acquire fence. Hence there is no existing acquire fence event correspond to any certificate step which can be referred later in the simulation step. As a result, this case is not possible.

Case SC FENCE F_{sc}:
In the event structure we extend the event structure G to G'. We extend the cover set $\mathbb{S}_i$ as well as the relations (spo, srf, smo) to $\mathbb{S}'_i$ along with the respective relations (spo$'$, srf', smo') by including an event e' where
(1) $\mathrm{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
(2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
(3) $\mathit{labels}(\mathrm{sequence}_{G.\mathrm{po}}(\mathbb{S}_i)).(e'.\mathrm{lab}) \in \mathbb{P}(i)$.

In this case the promise machine is updated as follows.
$\mathsf{M}' = \mathsf{M}$, $\mathcal{S}' = \{(x, t) \mid x \in \mathrm{Locs} \wedge \max(\mathcal{TS}(i).V.\mathrm{cur}(x), t') \wedge (x, t') \in \mathcal{S}\}$, and
$\mathcal{TS}' = \mathcal{TS}[i \mapsto \langle\langle \mathbb{P}(i), \mathit{labels}(\mathrm{sequence}_{\mathrm{spo}'}(\mathbb{S}'_i))\rangle, \mathcal{S}', \mathcal{TS}(i).\mathsf{P}\rangle]$
Now we do a case analysis on whether such an sc fence event e' exists in G or we append a new event.

Subcase $\nexists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i)$. $\mathsf{dom}(G.\mathsf{po};[\{e'\}]) \subseteq \mathbb{S}_i \wedge e'.\mathsf{lab} = \mathsf{F}_{\mathbf{sc}}$**:**
We create e' such that $e'.\mathsf{lab} = \mathcal{F}_{\mathsf{sc}}$ and append e' to event structure G to create G'. Then,

- $G'.\mathsf{E} = G.\mathsf{E} \uplus \{e' \mid e'.\mathsf{lab} = \mathcal{F}_{\mathsf{sc}}\}$ $G'.\mathsf{po} = G.\mathsf{po} \cup \{(e, e') \mid e \in (\mathbb{S}_i \cup \mathbb{S}_0)\}$
- $G'.\mathsf{jf} = G.\mathsf{jf}$
- $G'.\mathsf{ew} = G.\mathsf{ew}$

Let: $\mathbb{W}' \triangleq \mathbb{W}$.
Based on $\mathbb{W}'$, we derive following definitions in MS'.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathsf{mo}' \triangleq \mathsf{mo}$
- $\mathsf{sc}' \triangleq \mathsf{sc} \uplus \{(a, e') \mid a \in (G.\mathcal{F}_{\mathsf{sc}} \cap \mathbb{S})\}$
- $\mathsf{spo}' \triangleq (\mathsf{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathsf{srf}' \triangleq \mathsf{srf}$

Note that there may be incoming synchronization edges to the acquire fence, that is, $\mathsf{ssw} \subsetneq \mathsf{ssw}'$ and hence $\mathsf{shb} \subseteq \mathsf{shb}'$.
Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: G' *is consistent in* WEAKEST *model.*

- (CF) The constraint is preserved in G'. The argument is analogous to the scenario when we append a new $\mathsf{Ld}_o(x, v)$.
- (CFJ) Constraint (CFJ) is preserved in G'. The argument is analogous to the scenario when we append a new $\mathsf{St}_o(x, v)$.
- (VISJ) Constraint (VISJ) is preserved in G' as $G'.\mathsf{jf} = G.\mathsf{jf}$ and G satisfies constraint (VISJ).
- (ICF)
  We know that G satisfies (ICF). Suppose there exists an event $e_1 \in G$ which is in immediate conflict with e' in G', that is $G'. \sim (e_1, e')$ holds.
  Then (1) $\mathsf{dom}(G.\mathsf{po};[\{e_1\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e_1 \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e_1.\mathsf{lab}) \in \mathbb{P}(i)$.
  However, from definition of e' we already know that
  (1) $\mathsf{dom}(G.\mathsf{po};[\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e'.\mathsf{lab}) \in \mathbb{P}(i)$.
  Hence following the determinacy condition we know either $e_1 = e'$ or there exists no such e_1.
  Hence (ICF) is preserved in G'.
  Note. This was similar to the scenario when we append a new $\mathsf{F}_{\mathsf{REL}}(x, v)$.
- (ICFJ) Constraint (ICFJ) is preserved in G' as $e' \notin \mathcal{R}$ and G satisfies constraint (ICFJ).
- (COH) We know G preserves (COH) constraint, that is, $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic. The incoming edges to event e' are $G'.\mathsf{po}$ and $G'.\mathsf{hb}$ (due to $G'.\mathsf{sw}$ edges), and there is no outgoing edge concerning $G'.\mathsf{hb}$ or $G'.\mathsf{eco}_{\mathsf{strong}}$. As a result, $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic and G' preserves (COH) constraint.

(2) Condition to show: *The local state of each thread in* MS' *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*

In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.

For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS′ and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo′ we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\cdot e'.\mathsf{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma\cdot e'.\mathsf{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS′ and G'.

(3) Condition to show: *Whenever $\mathbb{W}'$ maps an event of G' to a message in* MS′*, then the location accessed and the written values match.*

We know that the event to message mappings for existing events in G.E and messages M do not change, that is, $\forall e \in G'.\mathsf{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$. If $e = e'$ then $\mathbb{W}'(e') = \perp$.
Hence $\mathbb{W}'$ preserves the condition.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$*, there are corresponding write events in G' that are* po*-after $\mathbb{S}'$.*

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS′, that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

(5) Condition to show: *For every location ℓ and thread j, the thread view of ℓ in the promise state* MS′ *records the timestamp of the maximal write visible to the covered events in G' of thread j.*

Essentially we have to show
$\forall j, \ell.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_j])\}$.
We know the relation holds in G.
For $j \neq i$, it is trivial because $\mathcal{TS}'.V(\ell) = \mathcal{TS}.V(\ell)$.
For $j = i$, we know that for a given location x,
$\mathcal{TS}'(i).V(x)$ extends $\mathcal{TS}(i).V(x)$ by choosing between timestamp from $\mathcal{TS}(i).V(x)$ and timestamp from $\mathsf{MS}_c.\mathcal{TS}'(c.\mathsf{tid}).V(x)$ where $\mathsf{imm}(\mathsf{sc}')(c, e')$ holds.
Hence $\forall \ell.\ \mathcal{TS}'(i).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$ holds.
Thus the relation holds between MS′ and G'.

(6) Condition to show: *The $\mathbb{S}'$ events in G' preserve coherence:* $\mathsf{shb}'; \mathsf{seco}'^?$ *is irreflexive.*
We know $\mathsf{shb}; \mathsf{seco}^?$ is irreflexive.
Following the definition of components of shb' and $\mathsf{seco}'^?$ we know $\mathsf{shb}'; \mathsf{seco}'^?$ is irreflexive.

(7) Condition to show: *The atomicity condition for update operations holds for $\mathbb{S}'$ events in G'.*

The argument is analogous to the case when we append a new $\mathsf{F}_{\mathsf{REL}}$.

(8) Condition to show: *The* SC *fences in G' are appropriately ordered by* sc′*.*

There is no outgoing edge from e' to any event in $\mathbb{S}'$.
Hence event e' cannot introduce a new $(\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$ path between two SC fences.
Hence $[G'.\mathcal{F}_{\mathsf{SC}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathcal{F}_{\mathsf{SC}}]$ implies $[G.\mathcal{F}_{\mathsf{SC}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathcal{F}_{\mathsf{SC}}]$.
We also know $\mathsf{sc} \subset \mathsf{sc}'$.
We also know $[G.\mathcal{F}_{\mathsf{SC}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathcal{F}_{\mathsf{SC}}] \subseteq \mathsf{sc}$.
Hence $[G'.\mathcal{F}_{\mathsf{SC}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathcal{F}_{\mathsf{SC}}] \subseteq \mathsf{sc}'$ holds.

(9) Condition to show: *The behavior of* MS′ *matches that of the* $\mathbb{S}'$ *events in* G'.

The argument is analogous to the case when we append a new F_{REL}.

Subcase $\exists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge e'.\mathsf{lab} = \mathsf{F}_{\mathbf{sc}}$**:**

Note that promising semantics does not promise over an SC fence. As a result, the certificate steps do not have any SC fence. Hence there is no existing SC fence event correspond to any certificate step which can be referred later in the simulation step. As a result, this case is not possible.

Case **Fulfill** op $=$ fulfill(m')**:**

In the event structure we extend the event structure G to G'. We extend the cover set $\mathbb{S}_i$ as well as the relations (spo, srf, smo) to $\mathbb{S}'_i$ along with the respective relations (spo′, srf′, smo′) by including a write (store or update) event e' where

(1) $\mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
(2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
(3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e'.\mathsf{lab}) \in \mathbb{P}(i)$.

In the promise machine let $m' = \langle x : v'@(f, t], -\rangle$.
Then the promise machine is updated as follows.
$\mathsf{M}' = \mathsf{M} \setminus \{m'\}, \mathcal{S}' = \mathcal{S}$,
and $\mathcal{TS}' = \mathcal{TS}[i \mapsto \langle\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle, V', \mathcal{TS}(i).\mathsf{P} \setminus \{m'\}\rangle]$
where $V' = \mathcal{TS}(i).V[x \mapsto t]$.

Now we do a case analysis on whether such an event e' exists in G or we append a new event. Based on $\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$ the event is either a store or an update event.

Subcase $\nexists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge (e'.\mathsf{lab} = \mathsf{St}_o(x, v') \vee (e'.\mathsf{lab} = \mathsf{U}_o(x, v, v') \wedge G.\mathsf{jf}(w_m, e')))$ **where** $\mathsf{wm} = \mathbb{W}(w_m)$**:**

We create e' such that $e'.\mathsf{lab} = \mathsf{St}_o(x, v')$ or $e'.\mathsf{lab} = \mathsf{U}_o(x, v, v')$ accordingly and append e' to event structure G to create G'. Then,

- $G'.\mathsf{E} = G.\mathsf{E} \uplus \{e'\}$
- $G'.\mathsf{po} = (G.\mathsf{po} \cup \{(e, e') \mid e \in (\mathbb{S}_i \cup \mathbb{S}_0)\})^+$
- $G'.\mathsf{jf} = G'.\mathsf{jf}G.\mathsf{jf} \uplus \{(w_m, e') \mid e' \in \mathsf{U} \wedge w_m \in G.\mathcal{W}_x \wedge w.\mathsf{wval} = v \wedge \mathbb{W}(w_m) = m\}$
- $G'.\mathsf{ew} = G.\mathsf{ew} \uplus \{(w_p, e') \mid w_p.\mathsf{id} \neq e'.\mathsf{id} \wedge \mathbb{W}(w_p) = m'\}$

Let: $\mathbb{W}' \triangleq \mathbb{W}[e' \mapsto m']$.

Based on $\mathbb{W}'$, we derive following definitions in MS′.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathsf{mo}' \triangleq \mathsf{mo} \uplus \{(a, e') \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(a).ts < \mathbb{W}'(e').ts\}$
  $\uplus \{(e', a) \mid a \in G.\mathcal{W}_x \wedge \mathbb{W}(a) \neq \perp \wedge \mathbb{W}'(e').ts < \mathbb{W}'(a).ts\}$
- $\mathsf{sc}' \triangleq \mathsf{sc}$
- $\mathsf{spo}' \triangleq (\mathsf{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathsf{srf}' \triangleq \mathsf{srf} \uplus \{(e', r) \mid (e', r) \in G'.\mathsf{rf}(e', r) \wedge r \in \mathbb{S}'\}$
  $\uplus \{(w_m, e') \mid e' \in G'.\mathsf{U} \wedge G'.\mathsf{rf}(w_m, e') \wedge w_m \in \mathbb{S}' \wedge w_m.\mathsf{wval} = v \wedge \mathbb{W}'(w_m) = \mathsf{wm}\}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: G' *is consistent in* WEAKEST *model.*

- (CF)
  We know that G satisfies (CF).
  New $G'.\mathsf{hb}$ edges are created by the incoming edges to e'. The outgoing $G'.\mathsf{rf}$ edge from e' does not result in any new synchronization.

The constraint is preserved in G'. If $e' \in G'.\mathsf{St}$ then the argument is analogous to the scenario when we append a new $\mathsf{St}_o(x, v)$ event. If $e' \in G'.\mathsf{U}$ then the argument is analogous to the scenario when we append a new $\mathsf{U}_o(x, v, v')$ event.
Hence G' satisfies (CF).

- (CFJ)
We know that G satisfies (CFJ).
Hence the new hb edges are created by the incoming edges to e'. The outgoing $G'.\mathsf{rf}$ edge from e' does not result in any new synchronization.
In that case the (CFJ) constraint is preserved in G'. If $e' \in G'.\mathsf{St}$ then the argument is analogous to the scenario when we append a new $\mathsf{St}_o(x, v)$ event. If $e' \in G'.\mathsf{U}$ then the argument is analogous to the scenario when we append a new $\mathsf{U}_o(x, v, v')$ event.
- (VISJ)
  - **case** $e' = \mathsf{St}_o(x, v')$.
  Constraint (VISJ) is preserved in G' as $G'.\mathsf{jf} = G.\mathsf{jf}$ and G satisfies constraint (VISJ).
  Note. This was same as the other scenario when we append a new $\mathsf{St}_o(x, v')$.
  - **case** $e' = \mathsf{U}_o(x, v, v')$.
  We study the possible cases of w_m.
    * If $G'.\mathsf{po}(w_m, e')$ then the condition holds as $(w_m, e') \notin G'.\mathsf{jfe}$.
    * We will show that G' satisfies (CFJ) constraint. Hence w_m cannot be in conflict with e', that is, $(w_m, e') \notin G'.\mathsf{cf}$.
    * w_m is in different thread and $G'.\mathsf{jfe}(w_m, e')$ holds. We know that $G \sim_{\{i\}} \mathsf{MS}$ and the simulation rules ensures that there is no *invisible* event in the $(\mathsf{T} \setminus \{i\})$ threads. Hence w_m is a visible event in G as well as in G'.

  Considering the above mentioned cases $G'.\mathsf{jfe}(w_m, e') \implies w_m \in \mathsf{vis}(G')$ holds and G' satisfies (VISJ) constraint.
  Note. This was same as the other scenario when we append a new $\mathsf{U}_o(x, v, v')$.
- (ICF) Constraint (ICF) is preserved in G. Now considering the cases of e':
  - **case** $e' = \mathsf{St}_o(x, v')$.
  Suppose there exists an event $e_1 \in G$ which is in immediate conflict with e' in G', that is $G'. \sim (e_1, e')$ holds.
  Then (1) $\mathsf{dom}(G.\mathsf{po}; [\{e_1\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e_1 \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e_1.\mathsf{lab}) \in \mathbb{P}(i)$.
  However, from definition of e' we already know that
  (1) $\mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i$,
  (2) $e' \in \mathbb{S}'_i \setminus \mathbb{S}_i$, and
  (3) $\mathit{labels}(\mathsf{sequence}_{G.\mathsf{po}}(\mathbb{S}_i)).(e'.\mathsf{lab}) \in \mathbb{P}(i)$.
  Hence following the determinacy condition we know either $e_1 = e'$ or there exists no such e_1.
  Hence (ICF) is preserved in G'.
  - **case** $e' = \mathsf{U}_o(x, v, v')$.
  Following the construction $e' \in G'.\mathcal{R}$ and following the determinacy condition,
  if $G'. \sim (e_1, e')$ then $e_1 \in \mathsf{Ld}$ or $e_1 \in \mathsf{U}$. Thus $(e_1, e') \in (G'.\mathcal{R} \times G'.\mathcal{R})$ and hence G' satisfies (ICF).
- (ICFJ) From the construction we know either $e' \in \mathsf{St}$ or there exists no e_1 such that $\mathsf{imm}(\mathsf{cf})(e_1, e')$ and $G.\mathsf{rf}(\mathbb{W}^{-1}(wm), e_1)$. Moreover, G satisfies constraint (ICFJ). As a result, G' satisfies (ICFJ).

- (COH) We know G preserves (COH) constraint, that is, $(G.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}})$ is acyclic.
  Now we check if G' has $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.
  If there exists $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle then the cycle contains $G'.\mathsf{rf}(e', r)$ and $(r, e') \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ holds.
  Since $(r, e') \notin G'.\mathsf{hb}$, $(r, e') \in (G'.\mathsf{hb}; G'.\mathsf{eco}_{\mathsf{strong}})$.
  Now we consider the cases of event e'.
  - **case** $e' = \mathsf{St}_o(x, v')$.
    The incoming edges to event e' are $G'.\mathsf{ew}$, $G'.\mathsf{hb}$, $G'.\mathsf{fr}_{\mathsf{strong}}$ edges and the outgoing edges are $G'.\mathsf{ew}$, $G'.\mathsf{rf}$ edges.
    Note that as e' is a newly appended event and no read event reads from e' no new $G'.\mathsf{rf}(w_m, -)$ is created.
    In that case the incoming edge to e' is $G'.\mathsf{fr}_{\mathsf{strong}}$ or $G'.\mathsf{mo}_{\mathsf{strong}}$.
    * **subcase** $G'.\mathsf{mo}_{\mathsf{strong}}$. Let $G'.\mathsf{mo}_{\mathsf{strong}}(w_1, e')$ be the incoming edge. In that case, considering Lemma 3, $\mathbb{W}'(w_m).ts < \mathbb{W}'(w_1).ts$, $\mathbb{W}'(w').ts < \mathbb{W}'(e').ts$. However, we know $\mathbb{W}'(w_m).ts = m'.ts = \mathbb{W}'(e').ts$. Hence this is not possible.
    * **subcase** $G'.\mathsf{fr}_{\mathsf{strong}}$. Let $G'.\mathsf{fr}_{\mathsf{strong}}(r_1, e')$ be the incoming edge.
      Let $G'.\mathsf{jf}(w_1, r_1)$ holds. In that case $G'.\mathsf{mo}_{\mathsf{strong}}(w_1, e')$ holds and hence like the earlier case $\mathbb{W}'(w_1).ts < m'.ts$ holds.
      However, we know that $(r, r_1) \in G'.\mathsf{hb}; G.\mathsf{eco}^?_{\mathsf{strong}}$ and hence following Lemma 3, $m'.ts \leq \mathbb{W}'(w_1).ts$. Hence a contradiction. As a result, $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is irreflexive.
  - **case** $e' = \mathsf{U}_o(x, v, v')$.
    The incoming edges to event e' are $G'.\mathsf{ew}$, $G'.\mathsf{hb}$, $G'.\mathsf{fr}_{\mathsf{strong}}$, and $G'.\mathsf{rf}$ edges and the outgoing edges are $G'.\mathsf{ew}$, $G'.\mathsf{rf}$ edges.
    Note that as e' is a newly appended event and no read event reads from e' no new $G'.\mathsf{rf}(w_m, -)$ is created.
    The argument for incoming $G'.\mathsf{ew}$, $G'.\mathsf{hb}$, $G'.\mathsf{fr}_{\mathsf{strong}}$ edges are same as the earlier cases where e' is a store event.
    So now we consider the case where $G'.\mathsf{rf}(-, e')$ is the incoming edge to e'. Let the edge be $G'.\mathsf{rf}(w'', e')$ and hence $(r, w'') \in (G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$.
    Following Lemma 3,
    (1) $m'.ts \leq \mathbb{W}'(w'').ts$. However, following the promising semantics for update operation we know that (2) $\mathbb{W}'(e'.ts > \mathbb{W}'(w'').ts)$ holds which implies $m'.ts > \mathbb{W}'(w'').ts$.
    The (1) and (2) contradicts and hence there is no $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ cycle.
    Hence $(G'.\mathsf{hb}; G'.\mathsf{eco}^?_{\mathsf{strong}})$ is irreflexive.

  Thus G' satisfies (COH).

As a result, G' is consistent in WEAKEST model.

(2) Condition to show: *The local state of each thread in* MS′ *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*
In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$.
We know that the relation holds between MS and G.
For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS′ and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo′ we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\cdot e'.\mathsf{lab}\rangle$

$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma \cdot e'.\mathsf{lab} \rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS' and G'.

(3) Condition to show: *Whenever $\mathbb{W}'$ maps an event of G' to a message in MS', then the location accessed and the written values match.*

We know that the event to message mappings for existing events in $G.\mathsf{E}$ and messages M do not change.

$$\forall e \in G'.\mathsf{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$$

If $e = e'$ then $\mathbb{W}'(e') = m'$ and $e'.\mathsf{loc} = m'.\mathsf{loc} = x$ and $e'.\mathsf{wval} = m'.\mathsf{wval} = v'$.
Hence $\mathbb{W}'$ preserves the condition.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$, *there are corresponding write events in G' that are* po*-after* $\mathbb{S}'$.

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS', that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

(5) Condition to show: *For every location ℓ and thread j, the thread view of ℓ in the promise state MS' records the timestamp of the maximal write visible to the covered events in G' of thread j.*

Essentially we have to show
$\forall j, \ell.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_j])\}$.
For $j \neq i$ or $j = i \wedge \ell \neq x$, it is trivial because $\mathcal{TS}'.V(\ell) = \mathcal{TS}.V(\ell)$.
For $j = i \wedge \ell = x$,
Based on the type of event e'

- **case** $e' \in G.\mathsf{St}_x$,
  following the promising semantics $\mathbb{W}'(e') = m'$, $m'.ts$ extends the view on x in thread i, and hence $\mathcal{TS}(i).V(x) < \mathcal{TS}'(i).V(x)$.
  In this case, $e' \in \mathsf{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_j])$.
  So $\mathcal{TS}'(i).V(x) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$ holds.
- **case** $e' \in G.\mathsf{U}_x$,
  Then, $\mathcal{TS}(i).V(x) = \max\{\mathbb{W}(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G.\mathsf{jf}^?; \mathsf{shb}^?; \mathsf{sc}^?; \mathsf{shb}^?; [\mathbb{S}_i])\}$ holds.
  Following the promising semantics, we know $\mathcal{TS}'(i).V(x)$ extends the thread view of x from $\mathcal{TS}(i).V(x)$ by reading from some message wm, and so $\mathcal{TS}(i).V(x) < \mathsf{wm}.ts$.
  Moreover, following the semantics of update in the promise machine, $\mathsf{wm}.ts < m'.ts$.
  So $\mathcal{TS}'(i).V(x) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$.

Thus the relation holds between MS' and G'.

(6) Condition to show: *The $\mathbb{S}'$ events in G' preserve coherence:* $\mathsf{shb}'; \mathsf{seco}'^?$ *is irreflexive.*
The argument is analogous to the new $\mathsf{St}_o(x, v, v')$ or new $\mathsf{U}_o(x, v, v')$ events.

(7) Condition to show: *The atomicity condition for update operations holds for $\mathbb{S}'$ events in G'.*
The argument is analogous to the new $\mathsf{St}_o(x, v, v')$ or new $\mathsf{U}_o(x, v, v')$ events.

(8) Condition to show: *The* SC *fences in G' are appropriately ordered by* sc'.

We know $[G.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}$ holds in G.
From definitions we know, $G'.\mathsf{F}_{\mathsf{sc}} = G.\mathsf{F}_{\mathsf{sc}}$, $\mathsf{sc}' = \mathsf{sc}$, $\mathsf{shb} \subseteq \mathsf{shb}'$, $\mathsf{seco} \subseteq \mathsf{seco}'$.
Consider a, b are two SC fences such that $(a, b) \in [G.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathsf{F}_{\mathsf{sc}}]$, and $\mathsf{sc}(a, b)$ holds.

In that case $(a, b) \in (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$ holds and $\mathsf{sc}'(a, b)$ holds.
To show $[G'.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$,
we have to show $(b, a) \notin (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
We show that by contradiction. Assume $(b, a) \in (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
This is possible due to the relations created to/from event e'.
Considering the relations in shb' and seco',
(1) when $e' \in G'.\mathsf{St}$, the incoming relations to event e' are shb', sfr', smo' and the outgoing edges are srf', smo'.
(2) when $e' \in G'.\mathsf{U}$, the incoming and outgoing relations to event e' are same as when $e' \in G'.\mathsf{St}$. Additionally, there are srf' incoming edges to e'.
In this case the path from b to a is $(b, e') \in \mathsf{shb}'; \mathsf{seco}'^?$,
and $(e', a) \in \mathsf{srf}'; \mathsf{seco}'^?; \mathsf{shb}'$ or $(e', a) \in \mathsf{smo}'; \mathsf{seco}'^?; \mathsf{shb}'$.
We analyze the cases of $(b, e') \in \mathsf{shb}'; \mathsf{seco}'^?$.
Similar to the new $\mathsf{St}_o(x, v, v')$ or the new $\mathsf{U}_o(x, v, v')$, in this case also $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) < \mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x)$ holds.
Now we consider the outgoing edges:

- $(e', a) \in \mathsf{srf}'; \mathsf{seco}'^?; \mathsf{shb}'$.
  There exists r such that $\mathsf{srf}'(e', a)$ and $(r, a) \in \mathsf{seco}'^?; \mathsf{shb}'$.
  Hence, $\mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x) = \mathsf{MS}_r.\mathcal{TS}(r.\mathsf{tid}).V(x) \leq \mathsf{MS}_a.\mathcal{TS}(a.\mathsf{tid}).V(x)$.
- $(e', a) \in \mathsf{smo}'; \mathsf{seco}'^?; \mathsf{shb}'$.
  There exists a write $w \in \mathbb{S}$ such that $\mathsf{smo}'(e', w)$ and $(w, a) \in \mathsf{seco}^?; \mathsf{shb}$.
  Hence, $\mathsf{MS}_{e'}.\mathcal{TS}(e'.\mathsf{tid}).V(x) < \mathsf{MS}_w.\mathcal{TS}(w.\mathsf{tid}).V(x) \leq \mathsf{MS}_a.\mathcal{TS}(a.\mathsf{tid}).V(x)$.

Considering both cases $\mathsf{MS}_b.\mathcal{TS}(b.\mathsf{tid}).V(x) < \mathsf{MS}_a.\mathcal{TS}(a.\mathsf{tid}).V(x)$ holds.
This is a contradiction and hence $(b, a) \notin (\mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}')$.
As a result, $[G'.\mathsf{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathsf{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$ holds.

(9) Condition to show: *The behavior of* MS' *matches that of the* $\mathbb{S}'$ *events in* G'.
The argument is analogous to the case when we append a new store or update event.

Subcase $\exists e' \in (G.\mathsf{E}_i \setminus \mathbb{S}_i).\ \mathsf{dom}(G.\mathsf{po}; [\{e'\}]) = \mathbb{S}_0 \cup \mathbb{S}_i \wedge (e'.\mathsf{lab} = \mathsf{St}_o(x, v') \vee (e'.\mathsf{lab} = \mathsf{U}_o(x, v, v') \wedge G.\mathsf{jf}(w_m, e')))$ **where** $\mathsf{wm} = \mathbb{W}(w_m)$**:**
In this case an event created for the promise certificate corresponds to the fulfill operation.
We take $G' = G$ and let $\mathbb{W}' = \mathbb{W}[e' \mapsto m']$ and
Based on $\mathbb{W}'$, we derive following definitions in MS'.

- $\mathbb{S}' \triangleq \mathbb{S} \uplus \{e'\}$
- $\mathsf{mo}' \triangleq \mathsf{mo}$
- $\mathsf{sc}' \triangleq \mathsf{sc}$
- $\mathsf{spo}' \triangleq (\mathsf{spo} \uplus \{(e, e') \mid e \in \mathbb{S}_0 \cup \mathbb{S}'_i\})^+$
- $\mathsf{srf}' \triangleq \mathsf{srf} \uplus \{(e', r) \mid (e', r) \in G'.\mathsf{rf}(e', r) \wedge r \in \mathbb{S}'\}$
  $\uplus \{(w_m, e') \mid e' \in G'.\mathsf{U} \wedge G'.\mathsf{rf}(w_m, e') \wedge w_m \mathbb{S}' \wedge w_m.\mathsf{wval} = v \wedge \mathbb{W}'(w_m) = \mathsf{wm}\}$

Now we check whether $G' \sim_{\{i\}} (\mathcal{TS}', \mathcal{S}', \mathsf{M}')$.

(1) Condition to show: G' *is consistent in* WEAKEST *model.*
G' is consistent as G is consistent in WEAKEST model.

(2) Condition to show: *The local state of each thread in* MS' *contains the program of that thread along with the sequence of covered events in* G' *of that thread.*
In this we have to show $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j)) \rangle$.
We know that the relation holds between MS and G.

For $j \neq i$, it is trivial because $\mathcal{TS}'(j) = \mathcal{TS}(j)$ holds from MS to MS′ and $\mathbb{S}'_j = \mathbb{S}_j$ holds from G to G'.
For $j = i$, we know $\mathcal{TS}(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$.
Hence following the definition of $\mathcal{TS}(i).\sigma$, $\mathbb{S}'_i$, spo′ we get
$\langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$
$= \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\cdot e'.\mathsf{lab}\rangle$
$= \langle \mathbb{P}(i), \mathcal{TS}(i).\sigma\cdot e'.\mathsf{lab}\rangle$
$= \mathcal{TS}'(i).\sigma$
Hence the condition is preserved between MS′ and G'.

(3) Condition to show: *Whenever $\mathbb{W}'$ maps an event of G' to a message in MS′, then the location accessed and the written values match.*

We know that the event to message mappings for existing events in G.E and messages M do not change.

$$\forall e \in G'.\mathsf{E}.\ e \neq e' \implies \mathbb{W}'(e) = \mathbb{W}(e)$$

If $e = e'$ then $\mathbb{W}'(e') = m'$ and $e'.\mathsf{loc} = m'.\mathsf{loc} = x$ and $e'.\mathsf{wval} = m'.\mathsf{wval} = v'$.
Hence $\mathbb{W}'$ preserves the condition.

(4) Condition to show: *For all outstanding promises of threads* $(\mathsf{T} \setminus \{i\})$*, there are corresponding write events in G' that are* po*-after $\mathbb{S}'$.*

We know that for each thread $j \neq i$ the set of promises are preserved from MS to MS′, that is, $\forall j \neq i.\ \mathcal{TS}(j).\mathsf{P} = \mathcal{TS}'(j).\mathsf{P}$.
We also know that G satisfies this condition.
Hence the condition is preserved in G'.

(5) Condition to show: *For every location ℓ and thread j, the thread view of ℓ in the promise state MS′ records the timestamp of the maximal write visible to the covered events in G' of thread j.*

The argument is analogous to the new $\mathsf{St}_o(x, v, v')$ or new $\mathsf{U}_o(x, v, v')$ events.

Thus the relation holds between MS′ and G'.

(6) Condition to show: *The $\mathbb{S}'$ events in G' preserve coherence:* $\mathsf{shb}';\mathsf{seco}'^{?}$ *is irreflexive.*
The argument is analogous to the case when we append a new store or update event for a fulfill operation.

(7) Condition to show: *The atomicity condition for update operations holds for $\mathbb{S}'$ events in G'.*
The argument is analogous to the new store or update event.

(8) Condition to show: *The* SC *fences in G' are appropriately ordered by* sc′.

The argument is analogous to the case when we append a new store or update event for a fulfill operation.

(9) Condition to show: *The behavior of* MS′ *matches that of the $\mathbb{S}'$ events in G'.*

The argument is analogous to the case when we append a new store or update event.

□

Now we prove Lemma 2.

Lemma 2. $G \sim \mathsf{MS} \land \mathsf{MS} \to \mathsf{MS}' \implies \exists G'.\ G \to_{\mathbb{P},\mathit{WEAKEST}}{}^{*}\ G' \land G' \sim \mathsf{MS}'$.

Proof. Following the promise machine step:

$$\frac{\begin{array}{c}\text{(MACHINE STEP)}\\ \langle \mathcal{TS}(i), \mathcal{S}, \mathsf{M}\rangle \xrightarrow{*} \langle TS', \mathcal{S}', \mathsf{M}'\rangle \qquad \langle TS', \mathcal{S}', \mathsf{M}'\rangle \xrightarrow{\mathsf{op}} \langle TS'', \mathcal{S}'', \mathsf{M}''\rangle \\ \langle TS'', \mathcal{S}'', \mathsf{M}''\rangle \text{ is consistent}\end{array}}{\langle \mathcal{TS}, \mathcal{S}, \mathsf{M}\rangle \xrightarrow{\mathsf{op}} \langle \mathcal{TS}[i \mapsto TS''], \mathcal{S}'', \mathsf{M}''\rangle}$$

Case analysis on the op:

$$\frac{\begin{array}{c}\text{(NP-STEP)}\\ \langle \mathcal{TS}(i), \mathcal{S}, \mathsf{M}\rangle \xrightarrow{np^+}_{\{i\}} \langle TS', \mathcal{S}', \mathsf{M}'\rangle \xrightarrow{np^*} \langle TS'', \mathcal{S}'', \mathsf{M}''\rangle \\ \mathsf{MS} = \langle \mathcal{TS}, \mathcal{S}, \mathsf{M}\rangle \qquad \mathsf{MS}' = \langle \mathcal{TS}[i \mapsto TS'], \mathcal{S}', \mathsf{M}'\rangle \qquad \mathsf{M}''.\mathsf{P} = \emptyset\end{array}}{\mathsf{MS} \xrightarrow{\mathsf{op}} \mathsf{MS}'}$$

$$\frac{\begin{array}{c}\text{(P-STEP)}\\ \langle \mathcal{TS}(i), \mathcal{S}, \mathsf{M}\rangle \xrightarrow{\mathsf{P}}_{\{i\}} \langle \mathcal{TS}(i), \mathcal{S}', \mathsf{M}'\rangle \xrightarrow{np^*} \langle TS'', \mathcal{S}'', \mathsf{M}''\rangle \\ \mathsf{MS} = \langle \mathcal{TS}, \mathcal{S}, \mathsf{M}\rangle \qquad \mathsf{MS}' = \langle \mathcal{TS}[i \mapsto TS'], \mathcal{S}', \mathsf{M}'\rangle \qquad \mathsf{M}''.\mathsf{P} = \emptyset\end{array}}{\mathsf{MS} \xrightarrow{\mathsf{op}} \mathsf{MS}'}$$

Case Non-promise step:

From $G \sim \mathsf{MS}$, we get $G \sim_{\{i\}} \mathsf{MS}$.

By Lemma 1 and induction, we have

$$\exists G'.\ G \rightarrow^* G' \wedge G' \sim_{\{i\}} \langle \mathcal{TS}[i \mapsto TS'], \mathcal{S}', \mathsf{M}'\rangle \tag{i}$$

and by Lemma 1 and induction, we have

$$\exists G''.\ G' \rightarrow^* G'' \wedge G'' \sim_{\{i\}} \langle \mathcal{TS}[i \mapsto TS''], \mathcal{S}'', \mathsf{M}''\rangle \tag{ii}$$

It remains to show $G'' \sim \mathsf{MS}'$.

We know that a certificate does not create any new message or SC fence. Hence $\mathsf{M}'' = \mathsf{M}'$ and $\mathcal{S}'' = \mathcal{S}'$.

We take $\mathbb{W}'' = \mathbb{W}'$ as there exists a write event in the certificate which maps to the promise message and in this case $\mathsf{mo}'' = \mathsf{mo}'$ and $\mathbb{S}'' = \mathbb{S}'$, $\mathsf{sc}'' = \mathsf{sc}'$, $\mathsf{spo}'' = \mathsf{spo}'$, $\mathsf{srf}'' = \mathsf{srf}'$, $\mathsf{seco}'' = \mathsf{seco}'$ hold.

(1) From Eq. (ii) we know that $G'' \sim_{\{i\}} \langle \mathcal{TS}[i \mapsto TS''], \mathcal{S}'', \mathsf{M}''\rangle$. Hence G'' is consistent.

(2) From Eq. (i) we know that
$\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$ holds.
Hence $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}''}(\mathbb{S}''_j))\rangle$ also holds since $\mathbb{S}'' = \mathbb{S}'$.

(3) From Eq. (i) we know $G'' \sim_{\{i\}} \langle \mathcal{TS}'[i \mapsto TS''], \mathcal{S}'', \mathsf{M}''\rangle$. We also know that $\mathsf{M}'' = \mathsf{M}'$ holds.
Hence whenever $\mathbb{W}''(e) = m$ then $e.\mathsf{loc} = m.\mathsf{loc}$ and $e.\mathsf{wval} = m.\mathsf{wval}$.

(4) From Eq. (i) we know $G' \sim_{\{i\}} \langle \mathcal{TS}[i \mapsto TS'], \mathcal{S}', \mathsf{M}'\rangle$. Hence the following also holds.
$\forall j \in (\mathsf{T} \setminus \{i\}).\ \forall e \in (\mathbb{S}'_0 \cup \mathbb{S}'_j).\ \mathcal{TS}'(j).\mathsf{P} \subseteq \{\mathbb{W}'(e') \mid (e, e') \in G'.\mathsf{po}\}$.
It implies

$$\forall j \in (\mathsf{T} \setminus \{i\}).\ \forall e \in (\mathbb{S}''_0 \cup \mathbb{S}''_j).\ \mathcal{TS}'(j).\mathsf{P} \subseteq \{\mathbb{W}''(e') \mid (e, e') \in G''.\mathsf{po}\} \tag{a}$$

In thread i events in $(\mathbb{S}'_0 \cup \mathbb{S}'_i)$ in G' has G'po-following events e' corresponding to the certificate of outstanding promises. Hence $\forall e \in (\mathbb{S}'_0 \cup \mathbb{S}'_i).\ \mathcal{TS}'(i).\mathsf{P} \subseteq \{\mathbb{W}'(e') \mid (e, e') \in G'.\mathsf{po}\}$.
It implies

$$\forall e \in (\mathbb{S}''_0 \cup \mathbb{S}''_i).\ \mathcal{TS}'(i).\mathsf{P} \subseteq \{\mathbb{W}''(e') \mid (e, e') \in G''.\mathsf{po}\} \tag{b}$$

Thus considering Eq. (a), Eq. (b) the following also holds

$$\forall j \in \mathsf{T}.\ \forall e \in (\mathbb{S}''_0 \cup \mathbb{S}''_j).\ \mathcal{TS}'(j).\mathsf{P} \subseteq \{\mathbb{W}''(e') \mid (e, e') \in G''.\mathsf{po}\}$$

Thus the condition is satisfied between G'' and MS'.

(5) From Eq. (i) we know

$$\forall i, x.\ \mathcal{TS}'(i).V(x) = \max\{\mathbb{W}(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$$

We know that $G'.\mathsf{po} \subseteq G''.\mathsf{po}$, $G'.\mathsf{jf} \subseteq G''.\mathsf{jf}$, $G'.\mathsf{ew} \subseteq G''.\mathsf{ew}$.
Hence from the definitions following holds:

$$\mathcal{TS}'(i).V(x) = \max\{\mathbb{W}''(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G''.\mathsf{jf}^?; \mathsf{shb}''^?; \mathsf{sc}''^?; \mathsf{shb}''^?; [\mathbb{S}''_i]\}$$

(6) From Eq. (ii) we already know $(\mathsf{shb}''; \mathsf{seco}''^?)$ is irreflexive.

(7) From Eq. (ii) we already know $[G''.\mathsf{U} \cap \mathbb{S}'']; (\mathsf{sfr}''; \mathsf{smo}'') = \emptyset$ holds.

(8) From Eq. (i) we know $[G'.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$.
From Eq. (ii) we know $[G''.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb}'' \cup \mathsf{shb}''; \mathsf{seco}''; \mathsf{shb}''; [G''.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}''$.
However, we know $\mathsf{sc}'' = \mathsf{sc}'$, $G''.\mathcal{F}_{\mathsf{sc}} = G'.\mathcal{F}_{\mathsf{sc}}$, and $\mathbb{S}'' = \mathbb{S}'$.
Hence $[G''.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb}'' \cup \mathsf{shb}''; \mathsf{seco}''; \mathsf{shb}''; [G''.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$.

(9) From Eq. (i) we know $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$.
From Eq. (ii) we know $\mathsf{Behavior}(\mathsf{MS}'') = \mathsf{Behavior}(G'', \mathbb{W}'', \mathbb{S}'')$.
However, $\mathsf{Behavior}(\mathsf{MS}'') = \mathsf{Behavior}(\mathsf{MS}')$ holds
and as a result, $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$.

As a result, $G'' \sim \mathsf{MS}'$ holds.

Case Promise step:

From $G \sim \mathsf{MS}$, we get $G \sim_{\{i\}} \mathsf{MS}$.
Also let $\mathsf{MS} \xrightarrow{\mathsf{op}}_i \mathsf{MS}'$ holds where $\mathsf{op} = \mathsf{promise}(m)$ in the thread i.
We show: $\exists G'.\ G \to^* G' \wedge G' \sim_{\{i\}} \mathsf{MS}'$
In this case $\mathcal{TS}' = \mathcal{TS}[i \mapsto TS']$, and $\mathsf{M}' = \mathsf{M} \uplus \{m\}$, and we take $G' = G$.
Thus it remains to show that $G \sim_{\{i\}} \mathsf{MS}'$.
We take $\mathbb{W}' = \mathbb{W}$
As a result $\mathsf{mo}' = \mathsf{mo}$ and $\mathbb{S}' = \mathbb{S}$, $\mathsf{sc}' = \mathsf{sc}$, $\mathsf{spo}' = \mathsf{spo}$, $\mathsf{srf}' = \mathsf{srf}$, $\mathsf{seco}' = \mathsf{seco}$ hold.

(1) From $G \sim \mathsf{MS}$ we know G is consistent and hence G' is also consistent.

(2) From $G' \sim_{\{i\}} \mathsf{MS}'$ we know that $\forall j \neq i.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j))\rangle$ holds.
Hence from the definitions $\forall j \neq i.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_j))\rangle$ also holds.
For $j = i$, $\mathcal{TS}'(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}'}(\mathbb{S}'_i))\rangle$ holds.
It implies, $\mathcal{TS}'(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$ also holds.
Hence $\forall j.\ \mathcal{TS}'(i).\sigma = \langle \mathbb{P}(i), \mathit{labels}(\mathsf{sequence}_{\mathsf{spo}}(\mathbb{S}_i))\rangle$ holds.
Thus the relation is preserved between G and MS'.

(3) From $G \sim \mathsf{MS}$ we know whenever $\mathbb{W}(m) = e$ then $e.\mathrm{loc} = m.\mathrm{loc}$ and $e.\mathrm{wval} = m.\mathrm{wval}$ holds. Since $\mathbb{W}' = \mathbb{W}$, the same also holds for $\mathbb{W}'$.

(4) We know $\forall j \in (\mathsf{T} \setminus \{i\}).\ \forall e \in (\mathbb{S}'_0 \cup \mathbb{S}'_j).\ \mathcal{TS}'(j).\mathsf{P} \subseteq \{\mathbb{W}'(e') \mid (e, e') \in G'.\mathrm{po}\}$.
Hence from the definitions $\forall j \in (\mathsf{T} \setminus \{i\}).\ \forall e \in (\mathbb{S}_0 \cup \mathbb{S}_j).\ \mathcal{TS}'(j).\mathsf{P} \subseteq \{\mathbb{W}(e) \mid (e, e) \in G.\mathrm{po}\}$ holds.

(5) From $G \sim_{\{i\}} \mathsf{MS}$ we know

$$\forall j \neq i.\ \mathcal{TS}(j).V(\ell) = \max\{\mathbb{W}(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_\ell]; G.\mathsf{jf}^?; \mathsf{shb}^?; \mathsf{sc}^?; \mathsf{shb}^?; [\mathbb{S}_j])\}$$

Since $G' = G$, $\mathbb{W}' = \mathbb{W}$, and $\mathcal{TS}' = \mathcal{TS}[i \mapsto TS']$ the following also holds.

$$\forall j \neq i.\ \mathcal{TS}'(j).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathrm{dom}([\mathcal{W}_\ell]; G.\mathsf{jf}^?; \mathsf{shb}^?; \mathsf{sc}^?; \mathsf{shb}^?; [\mathbb{S}_j])\}$$

(6) From $G \sim_{\{i\}} \mathsf{MS}$ we know $[G.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}$ holds.
We know $G'.\mathcal{F}_{\mathsf{sc}} = G.\mathcal{F}_{\mathsf{sc}}$, $\mathsf{shb}' = \mathsf{shb}$, $\mathsf{seco}' = \mathsf{seco}$, and $\mathsf{sc}' = \mathsf{sc}$.
Hence, $[G'.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$ also holds.

(7) From $G \sim_{\{i\}} \mathsf{MS}$ we know $(\mathsf{shb}; \mathsf{seco}^?)$ is irreflexive.
From the definition $\mathsf{shb}' = \mathsf{shb}$ and $\mathsf{seco}' = \mathsf{seco}$ hold.
Hence $(\mathsf{shb}'; \mathsf{seco}'^?)$ is irreflexive.

(8) From $G \sim_{\{i\}} \mathsf{MS}$ we know $[G.\mathsf{U} \cap \mathbb{S}]; (\mathsf{sfr}; \mathsf{smo}) = \emptyset$ holds.
We also know $\mathsf{sfr}' = \mathsf{sfr}$ and $\mathsf{smo}' = \mathsf{smo}$, $\mathbb{S}' = \mathbb{S}$, and $G.\mathsf{U} \subseteq G'.\mathsf{U}$.
Hence $[G'.\mathsf{U} \cap \mathbb{S}']; (\mathsf{sfr}'; \mathsf{smo}') = \emptyset$ also holds.

(9) From $G \sim_{\{i\}} \mathsf{MS}$ we know $\mathsf{Behavior}(\mathsf{MS}) = \mathsf{Behavior}(G, \mathbb{W}, \mathbb{S})$. We also know that $\mathbb{S}' = \mathbb{S}$ and $G' = G$.
Now following the definitions of MS' and G', we get $\mathsf{Behavior}(\mathsf{MS}) = \mathsf{Behavior}(\mathsf{MS}')$ and $\mathsf{Behavior}(G, \mathbb{W}, \mathbb{S}) = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$.
Hence $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$ holds.

Thus $G' \sim_{\{i\}} \mathsf{MS}'$ holds.

Subcase Certificate step following the promise step:

From $G' \sim \mathsf{MS}'$ we have $G' \sim_{\{i\}} \mathsf{MS}'$ and also the following holds.

$$\exists G''.\ G' \rightarrow^* G'' \wedge G'' \sim_{\{i\}} \mathsf{MS}'' = \langle \mathcal{TS}[i \mapsto TS''], \mathsf{M}'' \rangle$$

It remains to show $G'' \sim \mathsf{MS}'$

We know that $\mathcal{TS}'' = \mathcal{TS}'$. Moreover a certificate does not create any new message and hence $\mathsf{M}'' = \mathsf{M}'$.

We take $\mathbb{S}'' = \mathbb{S}'$, and $\mathbb{W}'' = \mathbb{W}'[e' \mapsto m]$ where $e'.\mathrm{loc} = m.\mathrm{loc}$, $e'.\mathrm{wval} = m.\mathrm{wval}$.

As a result, $\mathsf{mo}' \subseteq \mathsf{mo}''$, and $\mathbb{S}'' = \mathbb{S}'$, $\mathsf{sc}'' = \mathsf{sc}'$.

However, $e' \notin \mathbb{S}''$ and hence $\mathsf{smo}'' = \mathsf{smo}'$.

(1) We know that $G'' \sim_{\{i\}} \mathsf{MS}''$. Hence G'' is consistent.

(2) From $G' \sim \mathsf{MS}'$ we know that
$\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathrm{sequence}_{\mathsf{spo}'}(\mathbb{S}'_j)) \rangle$ holds.
We also know that $\mathbb{S}'' = \mathbb{S}'$ and $\mathcal{TS}'' = \mathcal{TS}'$.
Hence $\forall j.\ \mathcal{TS}'(j).\sigma = \langle \mathbb{P}(j), \mathit{labels}(\mathrm{sequence}_{\mathsf{spo}''}(\mathbb{S}''_j)) \rangle$ also holds.

(3) We know $G' \sim_{\{i\}} \mathsf{MS}'$. We also know that $\mathsf{M}'' = \mathsf{M}'$ holds.
Hence whenever $\mathbb{W}'(e) = m$, then $e.\mathrm{loc} = m.\mathrm{loc}$ and $e.\mathrm{wval} = m.\mathrm{wval}$ holds.

(4) We know $G' \sim_{\{i\}} \langle \mathcal{TS}[i \mapsto TS'], \mathcal{S}', \mathsf{M}' \rangle$. Hence the following also holds.
$\forall j \in (\mathsf{T} \setminus \{i\}).\ \forall e \in (\mathbb{S}'_0 \cup \mathbb{S}'_j).\ \mathcal{TS}'(j).\mathsf{P} \subseteq \{\mathbb{W}'(e') \mid (e, e') \in G'.\mathsf{po}\}$.
It implies

$$\forall j \in (\mathsf{T} \setminus \{i\}).\ \forall e \in (\mathbb{S}''_0 \cup \mathbb{S}''_j).\ \mathcal{TS}'(j).\mathsf{P} \subseteq \{\mathbb{W}''(e') \mid (e, e') \in G''.\mathsf{po}\} \tag{c}$$

In thread i events in $(\mathbb{S}'_0 \cup \mathbb{S}'_i)$ in G' has G'po-following events e' corresponding to the certificate of outstanding promises.
Hence $\forall e \in (\mathbb{S}'_0 \cup \mathbb{S}'_i).\ \mathcal{TS}'(i).\mathsf{P} \subseteq \{\mathbb{W}'(e') \mid (e, e') \in G'.\mathsf{po}\}$.
It implies

$$\forall e \in (\mathbb{S}''_0 \cup \mathbb{S}''_i).\ \mathcal{TS}'(i).\mathsf{P} \subseteq \{\mathbb{W}''(e') \mid (e, e') \in G''.\mathsf{po}\} \tag{d}$$

Thus considering Eq. (c), Eq. (d) the following also holds

$$\forall j \in \mathsf{T}.\ \forall e \in (\mathbb{S}''_0 \cup \mathbb{S}''_j).\ \mathcal{TS}'(j).\mathsf{P} \subseteq \{\mathbb{W}''(e') \mid (e, e') \in G''.\mathsf{po}\}$$

Thus the condition is satisfied between G'' and MS'.

(5) From $G' \sim_{\{i\}} \mathsf{MS}'$ We know

$$\mathcal{TS}'(i).V(\ell) = \max\{\mathbb{W}'(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_\ell]; G'.\mathsf{jf}^?; \mathsf{shb}'^?; \mathsf{sc}'^?; \mathsf{shb}'^?; [\mathbb{S}'_i])\}$$

We know that $G'.\mathsf{E} \subseteq G''.\mathsf{E}$, $G'.\mathsf{po} \subseteq G''.\mathsf{po}$, $G'.\mathsf{jf} \subseteq G''.\mathsf{jf}$, $G'.\mathsf{ew} \subseteq G''.\mathsf{ew}$, $\mathcal{TS}'' = \mathcal{TS}'$, $\mathbb{S}'' = \mathbb{S}'$, and $\mathbb{W}'' = \mathbb{W}'[e' \mapsto m]$.
Hence from the definitions following holds:

$$\mathcal{TS}'(i).V(x) = \max\{\mathbb{W}''(e).ts \mid e \in \mathsf{dom}([\mathcal{W}_x]; G''.\mathsf{jf}^?; \mathsf{shb}''^?; \mathsf{sc}''^?; \mathsf{shb}''^?; [\mathbb{S}''_i]\}$$

(6) We know $(\mathsf{shb}'; \mathsf{seco}'^?)$ is irreflexive.
From the definition $\mathsf{shb}'' = \mathsf{shb}'$ and $\mathsf{seco}'' = \mathsf{seco}'$.
Hence $(\mathsf{shb}''; \mathsf{seco}''^?)$ is irreflexive.

(7) From $G' \sim_{\{i\}} \mathsf{MS}'$ we know $[G'.\mathsf{U} \cap \mathbb{S}']; (\mathsf{sfr}'; \mathsf{smo}') = \emptyset$ holds.
We also know $\mathsf{sfr}'' = \mathsf{sfr}'$ and $\mathsf{smo}'' = \mathsf{smo}'$, $\mathbb{S}'' = \mathbb{S}'$, and $G'.\mathsf{U} \subseteq G''.\mathsf{U}$.
Hence $[G''.\mathsf{U} \cap \mathbb{S}'']; (\mathsf{sfr}''; \mathsf{smo}'') = \emptyset$ also holds.

(8) We know $\mathbb{S}'' = \mathbb{S}'$, $\mathsf{mo}' \subseteq \mathsf{mo}''$, $\mathsf{sc}'' = \mathsf{sc}'$.
We also know that $[G'.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb}' \cup \mathsf{shb}'; \mathsf{seco}'; \mathsf{shb}'; [G'.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}'$ holds.
Hence, $[G''.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb}'' \cup \mathsf{shb}''; \mathsf{seco}''; \mathsf{shb}''; [G''.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}''$ also holds.

(9) From $G' \sim_{\{i\}} \mathsf{MS}'$ we know $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$.
From $G'' \sim_{\{i\}} \mathsf{MS}''$ we know $\mathsf{Behavior}(\mathsf{MS}'') = \mathsf{Behavior}(G'', \mathbb{W}'', \mathbb{S}'')$.
From definitions $\mathsf{Behavior}(\mathsf{MS}'') = \mathsf{Behavior}(\mathsf{MS}')$
and $\mathsf{Behavior}(G'', \mathbb{W}'', \mathbb{S}'') = \mathsf{Behavior}(G', \mathbb{W}', \mathbb{S}')$ holds.
Hence $\mathsf{Behavior}(\mathsf{MS}') = \mathsf{Behavior}(G'', \mathbb{W}'', \mathbb{S}'')$ holds.

Hence $G'' \sim \mathsf{MS}'$ holds.

□

Finally we restate and prove Theorem 1.

Theorem 1. *For a program* $\mathbb{P}$, $\mathsf{Behavior}_{\mathsf{PS}}(\mathbb{P}) \subseteq \mathsf{Behavior}_{\textit{WEAKEST}}(\mathbb{P})$.

Formal statement:

$\forall \mathbb{P}.\ \forall \mathsf{MS}.\ (\mathsf{MS}_{\mathsf{init}}(\mathbb{P}) \rightarrow^* \mathsf{MS}\ \wedge\ \mathsf{MS} \not\rightarrow).\ \exists G, \mathsf{X}.\ G_{\mathsf{init}} \rightarrow_{\mathbb{P},\textsc{weakest}}{}^* G \wedge \mathsf{X} \in \mathsf{ex}_{\textsc{weakest}}(G).$
$\wedge \mathsf{Behavior}(\mathsf{MS}) = \mathsf{Behavior}(\mathsf{X})$

Proof. **Step 1.** Given a program $\mathbb{P}$, from Lemma 2 we show that using the simulation relation in Definition 6, we can follow the promise machine steps and for a promise machine state state MS we can construct an WEAKEST event structure G, that is, $G_{\mathsf{init}} \rightarrow_{\mathbb{P},\textsc{weakest}}{}^{*} G$.

Step 2. Now we extract a consistent execution X from G where $\mathsf{X} \in \mathsf{ex}_{\textsc{weakest}}(G)$, such that Behavior(MS) = Behavior(X).

Given the event structure G along with $\mathbb{S}$ and related sets,
the execution $\mathsf{X} = \langle \mathsf{E}, \mathsf{po}, \mathsf{rf}, \mathsf{mo}\rangle$ is as follows.

- $\mathsf{X.E} = \mathbb{S}$,
- $\mathsf{X.po} = \mathsf{spo}$,
- $\mathsf{X.rf} = \mathsf{srf}$, and
- $\mathsf{X.mo} = \mathsf{smo}$

Note that the events in X.E is conflict-free as $\mathbb{S}$ is conflict-free in G.
Now we check whether execution X is consistent.

- from the definitions of spo, srf, smo, we know
  $\mathsf{X.po} \subseteq (\mathbb{S} \times \mathbb{S})$, $\mathsf{X.rf} \subseteq (\mathbb{S} \times \mathbb{S})$, and $\mathsf{X.mo} \subseteq (\mathbb{S} \times \mathbb{S})$.
  Hence X is (Well-formed).
- From the definition, we know smo is total as the order on the timestamps on the same location is total in the promise machine.
  Hence X.mo is total and (total-MO) holds in X.
- From the construction of G we know that $\mathsf{shb}; \mathsf{seco}^{?}$ is irreflexive.
  Hence $(\mathsf{X.hb}_{\mathsf{C11}}; \mathsf{X.eco}^{?})$ is irreflexive and (Coherence) holds in G.
- From the construction we know that $[G.\mathsf{U} \cap \mathbb{S}]; (\mathsf{sfr}; \mathsf{smo}) = \emptyset$ holds. From the definition we know that $\mathsf{X.U} = (G.\mathsf{U} \cap \mathbb{S})$, $\mathsf{X.fr} = \mathsf{sfr}$, and also $\mathsf{X.mo} = \mathsf{smo}$ holds.
  Hence $[\mathsf{X.U}]; (\mathsf{X.fr}; \mathsf{X.mo}) = \emptyset$ hold and X preserves (Atomicity).
- From the simulation relation in the construction we know that sc is total in G and
  $[G.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathcal{F}_{\mathsf{sc}}] \subseteq \mathsf{sc}$ holds.
  Hence $[G.\mathcal{F}_{\mathsf{sc}}]; \mathsf{shb} \cup \mathsf{shb}; \mathsf{seco}; \mathsf{shb}; [G.\mathcal{F}_{\mathsf{sc}}]$ is irreflexive.
  From definition we know that $\mathsf{X}.\mathcal{F}_{\mathsf{sc}} = G.\mathcal{F}_{\mathsf{sc}}$, $\mathsf{X.hb}_{\mathsf{C11}} = \mathsf{shb}$, and $\mathsf{X.eco} = \mathsf{seco}$ hold.
  As a result, $\mathsf{X.psc}_{\mathsf{F}} = [\mathsf{X}.\mathcal{F}_{\mathsf{sc}}]; \mathsf{X.hb}_{\mathsf{C11}} \cup \mathsf{X.hb}_{\mathsf{C11}}; \mathsf{X.eco}; \mathsf{X.hb}_{\mathsf{C11}}; [\mathsf{X}.\mathcal{F}_{\mathsf{sc}}]$ is irreflexive.
  Note that X does not have any SC memory access and hence $\mathsf{X.psc}_{\mathsf{base}} = \emptyset$.
  Hence X preserves (SC).

Thus X is consistent and hence $\mathsf{X} \in \mathsf{ex}_{\textsc{weakest}}(G)$.
Step 3. From the construction we know that Behavior(MS) = Behavior(G, $\mathbb{W}$, $\mathbb{S}$).
Hence from the definitions Behavior(MS) = Behavior(X).

Thus considering step 1, 2, 3 the theorem holds. □

B CAUSALITY TEST CASES

$$\begin{array}{l} r_1 = X; \\ \text{if}(r_1 \geq 0)\ Y = 1; \end{array} \left\| \begin{array}{l} r_2 = Y; \\ X = r_2; \end{array} \right.$$

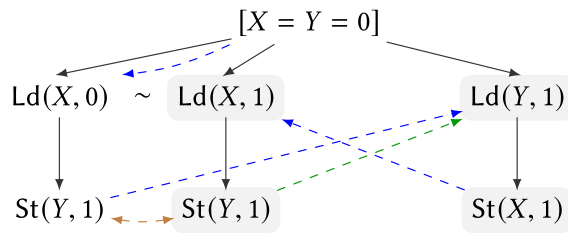


Fig. 14. Case 1. Allowed $r_1 == r_2 == 1$.

$$\begin{array}{l} r_1 = X; \\ r_2 = X; \\ \text{if}(r_1 == r_2) \\ \quad Y = 1; \end{array} \left\| \begin{array}{l} r_3 = Y; \\ X = r_3; \end{array} \right.$$

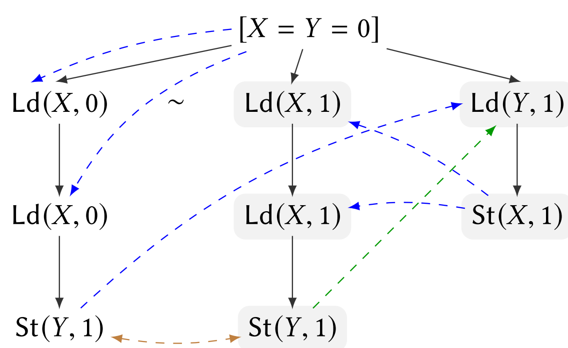


Fig. 15. Case 2. Allowed $r_1 == r_2 == r_3 == 1$.

$$\begin{array}{l} r_1 = X; \\ r_2 = X; \\ \text{if}(r_1 == r_2) \\ \quad Y = 1; \end{array} \left\| \begin{array}{l} r_3 = Y; \\ X = r_3; \end{array} \right\| X = 2;$$

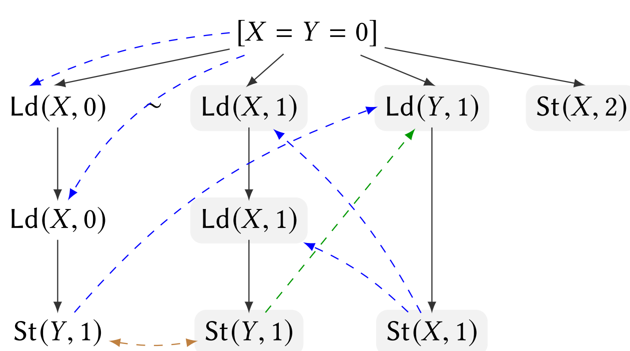


Fig. 16. Case 3. Allowed $r_1 == r_2 == r_3 == 1$.

$$\begin{array}{l||l} r_1 = X; & r_2 = Y; \\ Y = r_1; & X = r_2; \end{array}$$

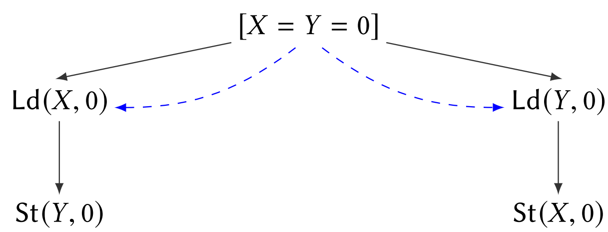


Fig. 17. Case 4. Forbidden $r_1 == r_2 == 1$.

$$\begin{array}{l||l||l||l} r_1 = X; & r_2 = Y; & r_3 = Z; & Z = 1; \\ Y = r_1; & X = r_2; & X = r_3; & \end{array}$$

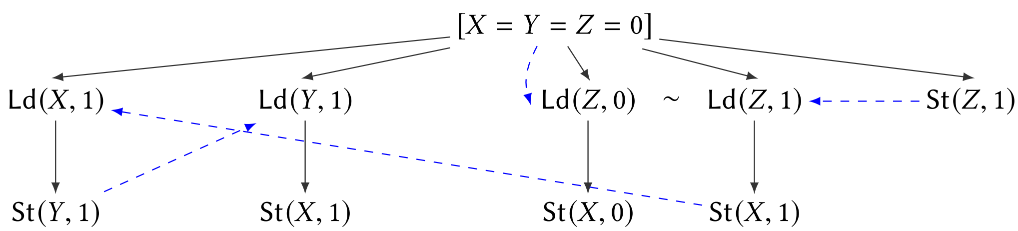


Fig. 18. Case 5. Forbidden $r_1 == r_2 == 1, r_3 == 0$. However, a sequence of transformations result this behavior.

$$\begin{array}{l||l} & r_2 = B; \\ r_1 = A; & \text{if}(r_2 == 1) \\ \text{if}(r_1 == 1) & \quad A = 1; \\ \quad B = 1; & \text{if}(r_2 == 0) \\ & \quad A = 1; \end{array}$$

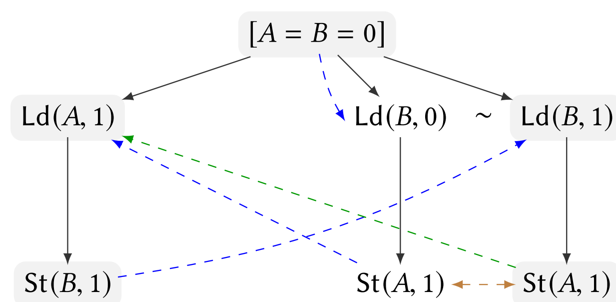


Fig. 19. Case 6. Allowed $r_1 == r_2 == 1$.

$$\begin{array}{l||l} r_1 = Z; & r_3 = Y; \\ r_2 = X; & Z = r_3; \\ Y = r_2; & X = 1; \end{array}$$

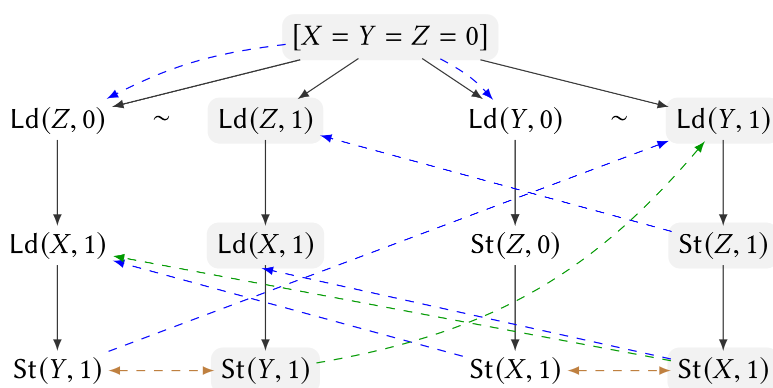


Fig. 20. Case 7. Allowed $r_1 == r_2 == r_3 == 1$.

$$
\begin{array}{l}
r_1 = X;\\
r_2 = 1 + r_1 * r_1 - r_1;\\
Y = r_2;
\end{array}
\;\Bigg\|\;
\begin{array}{l}
r_3 = Y;\\
X = r_3;
\end{array}
$$

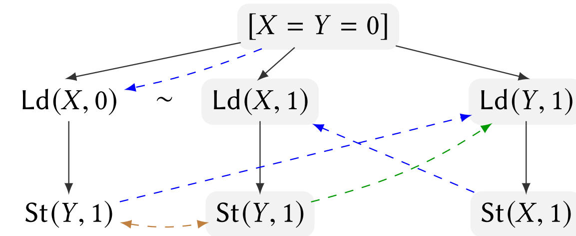


Fig. 21. Case 8. Allowed $r_1 == r_2 == 1$.

$$
\begin{array}{l}
r_1 = X;\\
r_2 = 1 + r_1 * r_1 - r_1;\\
Y = r_2;
\end{array}
\;\Bigg\|\;
\begin{array}{l}
r_3 = Y;\\
X = r_3;
\end{array}
\;\Bigg\|\;
X = 2;
$$

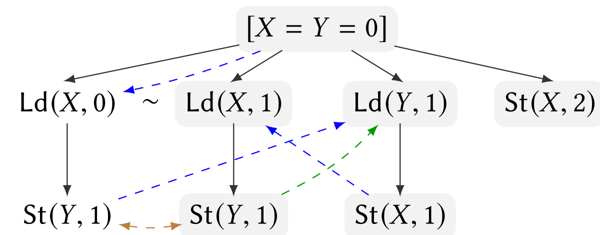


Fig. 22. Case 9. Allowed $r_1 == r_2 == 1$.

$$
\begin{array}{l}
r_1 = X;\\
r_2 = 1 + r_1 * r_1 - r_1;\\
Y = r_2;
\end{array}
\;\Bigg\|\;
\begin{array}{l}
r_3 = Y;\\
X = r_3;
\end{array}
\;\Bigg\|\;
X = 0;
$$

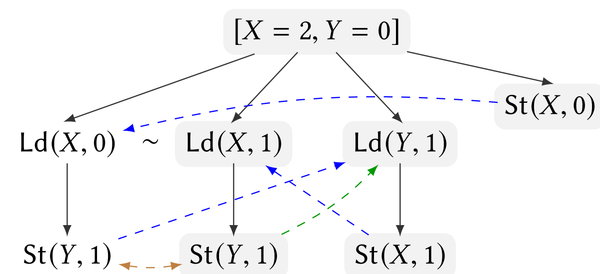


Fig. 23. Case 9a. Allowed $r_1 == r_2 == 1$.

$$
\begin{array}{l}
r_1 = X;\\
\mathrm{if}(r_1 == 1)\\
\qquad Y = 1;
\end{array}
\;\Bigg\|\;
\begin{array}{l}
r_2 = Y;\\
\mathrm{if}(r_2 == 1)\\
\qquad X = 1;
\end{array}
\;\Bigg\|\;
\begin{array}{l}
r_3 = Z;\\
\mathrm{if}(r_3 == 1)\\
\qquad X = 1;
\end{array}
\;\Bigg\|\;
Z = 1;
$$

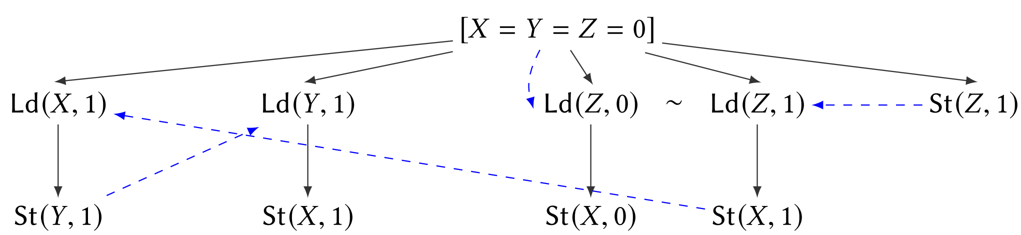


Fig. 24. Case 10. Forbidden $r_1 == r_2 == 1, r_3 == 0$. Same event structure as Fig. 18. imilar to test case 5, a sequence of transformations result this behavior.

$r_1 = Z;$	$r_4 = W;$
$W = r_1;$	$r_3 = Y;$
$r_2 = X;$	$Z = r_3;$
$Y = r_2;$	$X = 1;$

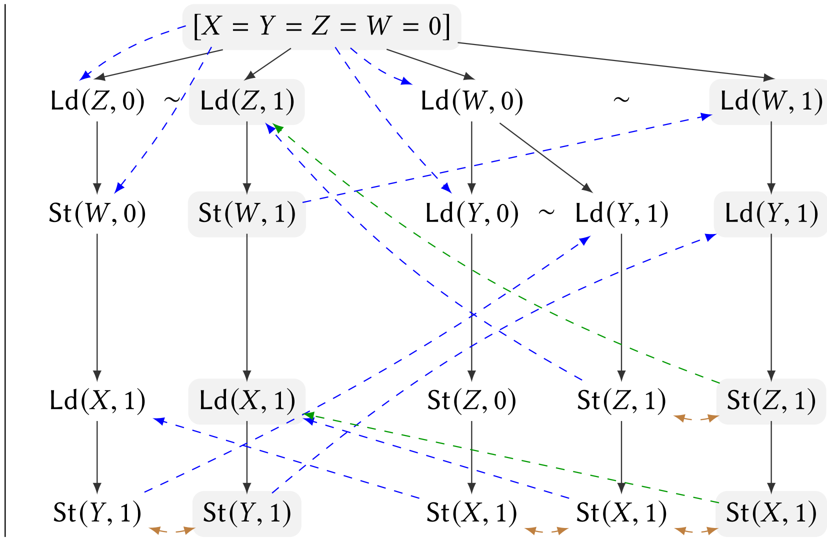


Fig. 25. Case 11. Allowed $r_1 == r_2 == r_3 == r_4 == 1$.

$X = Y = 0; a[0] = 1; a[1] = 2;$

$r_1 = X;$	
$a[r_1] = 0;$	$r_3 = Y;$
$r_2 = a[0];$	$X = r_3;$
$Y = r_2;$	

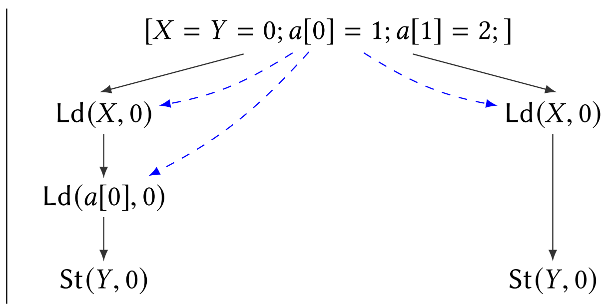


Fig. 26. Case 12. Forbids $r_1 == r_2 == r_3 == 1$.

$r_1 = X;$	$r_2 = Y;$
$\text{if}(r_1 == 1)$	$\text{if}(r_2 == 1)$
$\quad Y = 1;$	$\quad X = 1;$

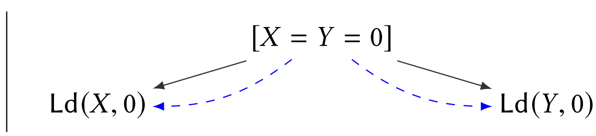


Fig. 27. Case 13. Forbids $r1 == r2 == 1$.

	do{
$r_1 = A;$	$\quad r_2 = Y_{\text{SC}};$
$\text{if}(r_1 == 0)$	$\quad r_3 = B;$
$\quad Y_{\text{SC}} = 1;$	$\}$ while$(r_2 + r_3 == 0);$
else	$A = 1;$
$\quad B = 1;$	

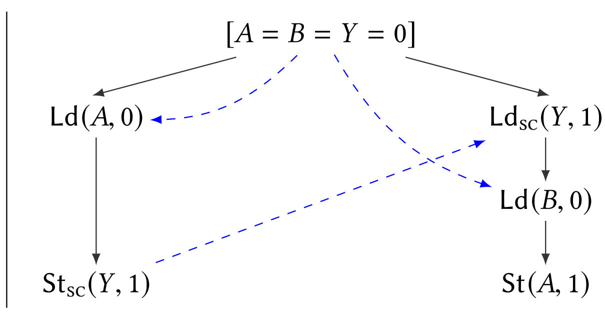


Fig. 28. Case 14. Forbids $r_1 = r_3 = 1; r_2 = 0$. In [Manson et al. 2004] Y is 'volatile' in Java. We map Java volatile to SC in C11 as the reordering rules are same.

$$
\begin{array}{l}
r_0 = X_{sc}; \\
\text{if}(r_0 == 1)\ \ r_1 = A; \\
\text{else}\ \ r_1 = 0; \\
\text{if}(r_1 == 0)\ \ Y_{sc} = 1; \\
\text{else}\ \ B = 1;
\end{array}
\;\middle\|\;
\begin{array}{l}
\text{do}\{ \\
\quad r_2 = Y_{sc};\ \ r_3 = B; \\
\}\ \text{while}(r_2 + r_3 == 0); \\
A = 1;
\end{array}
\;\middle\|\;
X_{sc} = 1;
$$

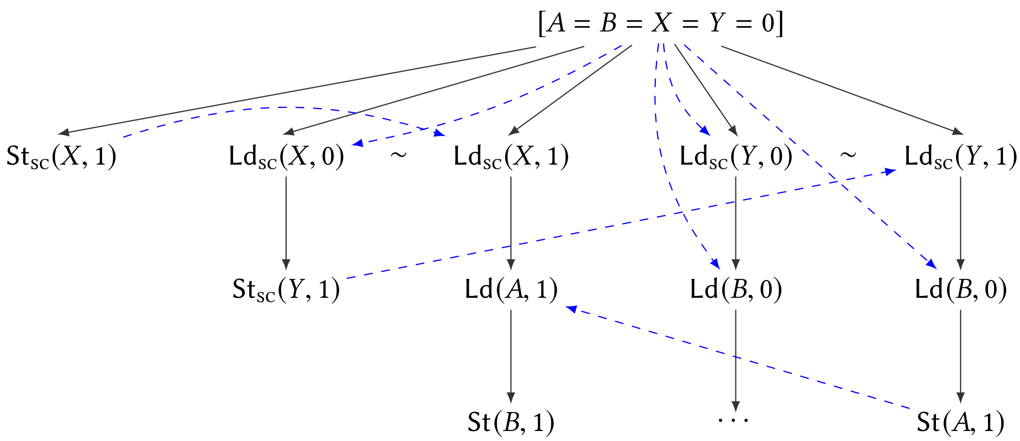


Fig. 29. Case 15. Forbids $r_1 == r_3 == 1; r_2 == 0$. In [Manson et al. 2004] X and Y are 'volatile' in Java. We map Java volatile to SC in C11 as the reordering rules are same.

$$
\begin{array}{l}
r_1 = X; \\
X = 1;
\end{array}
\middle\|
\begin{array}{l}
r_2 = X; \\
X = 2;
\end{array}
$$

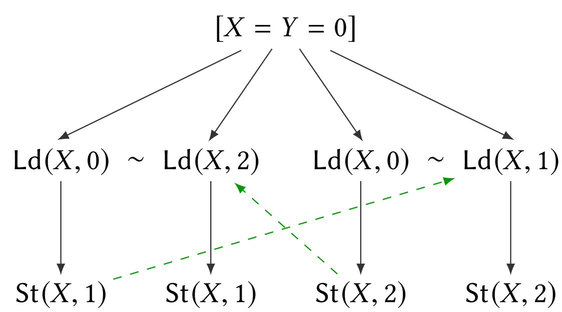


Fig. 30. Case 16. Behavior in question: $r_1 = 2, r_2 = 1$. This is allowed in Manson et al. [2004]. The behavior is allowed in basic event structure and in extracted execution as they do not enforce coherence. The WEAKEST model constructs an event structure with these events but disallows the incoherent behavior in the extracted execution. The WEAKESTMO model does not accommodate all these events together in any event structure and in cosequence disallows the incoherent behavior in the extracted execution.

```
r3 = X;           ||                 r3 = X;            ||
if(r3! = 4)       || r2 = Y;         if(r3 == 0)        || r2 = Y;
   X = 4;         || X = r2;            X = 4;          || X = r2;
r1 = X;           ||                 r1 = X;            ||
Y = r1;           ||                 Y = r1;            ||
```

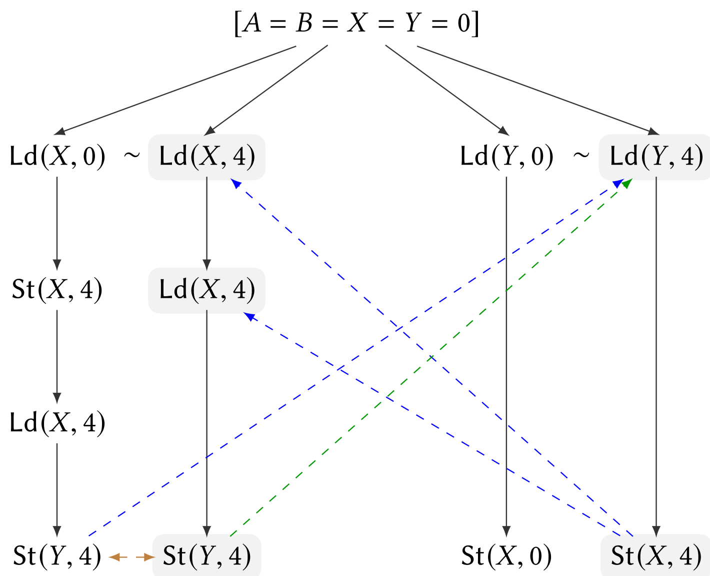


Fig. 31. Case 17 and 18. Allows $r_1 == r_2 == r_3 == 4$.

```
r1 = X; || r2 = Y; || r3 = X;             r1 = X; || r2 = Y; || r3 = X;
Y = r1; || X = r2; || if(r3! = 4)         Y = r1; || X = r2; || if(r3 == 0)
                   ||    X = 4;                               ||    X = 4;
```

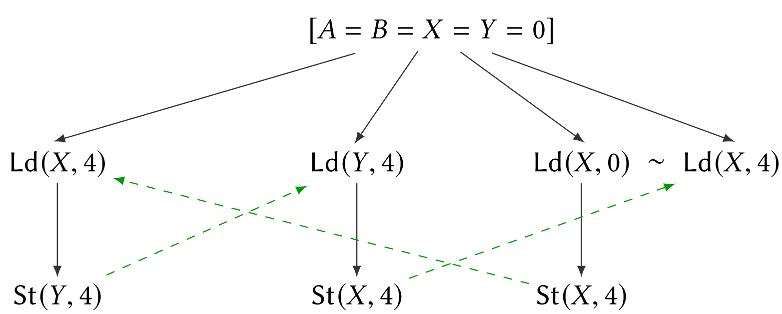


Fig. 32. Case 19 and 20. Event Structure Forbids $r_1 == r_2 == r_3 == 4$.

B.1 Allowing Forbidden Behaviors

Now we see certain behaviors which are disallowed by Manson et al. [2004] and our proposed scheme but are possible after a number of program transformations.

Testcase 5. The $r_1 == r_2 == 1, r_3 == 0$ outcome is possible after a sequence of transformations as follows.

$$
\begin{array}{ll}
 & \begin{array}{l} r_1 = X; \\ Y = r_1; \end{array} \Bigg\| \begin{array}{l} r_2 = Y; \\ X = r_2; \end{array} \Bigg\| \begin{array}{l} r_3 = Z; \\ X = r_3; \end{array} \Bigg\| \; Z = 1; \\
\rightsquigarrow & \begin{array}{l} r_1 = X; \\ Y = r_1; \end{array} \Bigg\| \begin{array}{l} r_2 = Y; \\ \text{if}(r_2 == 1)\ X = 1; \text{else}\ X = r_2; \end{array} \Bigg\| \begin{array}{l} r_3 = Z; \\ X = r_3; \end{array} \Bigg\| \; Z = 1; \\
\rightsquigarrow & \begin{array}{l} r_1 = X; \\ Y = r_1; \end{array} \Bigg\| \begin{array}{l} r_2 = Y; \\ \text{if}(r_2 == 1)\ X = 1; \text{else}\ X = r_2; \\ \{r_3 = Z; X = r_3;\} \mid\mid \{Z = 1;\} \end{array} \\
\rightsquigarrow & \begin{array}{l} r_1 = X; \\ Y = r_1; \end{array} \Bigg\| \begin{array}{l} r_2 = Y; \\ \text{if}(r_2 == 1)\{ \\ \ X = 1; \\ \ \{r_3 = Z; X = r_3;\} \mid\mid \{Z = 1;\} \\ \}\text{else}\{ \\ \ X = r_2; \\ \ \{r_3 = Z; X = r_3;\} \mid\mid \{Z = 1;\} \\ \} \end{array} \\
\rightsquigarrow & \begin{array}{l} r_1 = X; \\ Y = r_1; \end{array} \Bigg\| \begin{array}{l} r_2 = Y; \\ \text{if}(r_2 == 1)\ \{X = 1; r_3 = Z; X = r_3; Z = 1;\} \\ \text{else}\ \{X = r_2; Z = 1; r_3 = Z; X = r_3;\} \end{array} \\
\rightsquigarrow & \begin{array}{l} r_1 = X; \\ Y = r_1; \end{array} \Bigg\| \begin{array}{l} r_2 = Y; \\ \text{if}(r_2 == 1)\ \{X = 1; r_3 = Z; X = r_3; Z = 1;\} \\ \text{else}\ \{\cancel{X = r_2}; Z = 1; r_3 = 1; X = 1;\} \end{array} \\
\rightsquigarrow & \begin{array}{l} a : r_1 = X; \\ b : Y = r_1; \end{array} \Bigg\| \begin{array}{l} c : X = 1; \\ d : r_2 = Y; \\ \text{if}(r_2 == 1)\ \{e : r_3 = Z; X = r_3; Z = 1;\} \\ \text{else}\ \{Z = 1; r_3 = 1;\} \end{array}
\end{array}
$$

Now it is possible to have an interleaving c, a, b, d, e which results in $r_1 == r_2 == 1, r_3 == 0$.

Testcase 10. Similar to test case 5 the $r_1 == r_2 == 1, r_3 == 0$ outcome is possible after a sequence of transformations as follows.

$$
\begin{array}{l} r_1 = X; \\ \text{if}(r_1 == 1) \\ \quad Y = 1; \end{array} \Bigg\| \begin{array}{l} r_2 = Y; \\ \text{if}(r_2 == 1) \\ \quad X = 1; \end{array} \Bigg\| \begin{array}{l} r_3 = Z; \\ \text{if}(r_3 == 1) \\ \quad X = 1; \end{array} \Bigg\| \; Z = 1; \quad \rightsquigarrow
$$

$$
\begin{array}{l} r_1 = X; \\ \text{if}(r_1 == 1) \\ \quad Y = 1; \end{array} \Bigg\| \begin{array}{l} r_2 = Y; \\ \text{if}(r_2 == 1) \\ \quad X = 1; \\ \text{else} \\ \quad X = 0; \end{array} \Bigg\| \begin{array}{l} r_3 = Z; \\ \text{if}(r_3 == 1) \\ \quad X = 1; \end{array} \Bigg\| \; Z = 1; \quad \rightsquigarrow
$$

$$
\begin{array}{r||l}
 & r_2 = Y; \\
 & \mathsf{if}(r_2 == 1)\{ \\
 & \quad X = 1; \\
 & \quad r_3 = Z; \\
 & \quad \mathsf{if}(r_3 == 1) \\
 & \qquad X = 1; \\
r_1 = X; \quad\quad & \quad Z = 1; \\
\mathsf{if}(r_1 == 1) & \} \\
Y = 1; & \mathsf{else}\{ \\
 & \quad X = 0; \\
 & \quad Z = 1; \\
 & \quad r_3 = Z; \\
 & \quad \mathsf{if}(r_3 == 1) \\
 & \qquad X = 1; \\
 & \}
\end{array}
\quad \rightsquigarrow \quad
\begin{array}{r||l}
 & r_2 = Y; \\
 & \mathsf{if}(r_2 == 1)\{ \\
 & \quad X = 1; \\
 & \quad r_3 = Z; \\
 & \quad \mathsf{if}(r_3 == 1) \\
 & \qquad X = 1; \\
r_1 = X; \quad\quad & \quad Z = 1; \\
\mathsf{if}(r_1 == 1) & \} \\
Y = 1; & \mathsf{else}\{ \\
 & \quad X = 0; \\
 & \quad Z = 1; \\
 & \quad r_3 = 1; \\
 & \quad X = 1; \\
 & \}
\end{array}
\quad \rightsquigarrow
$$

$$
\begin{array}{r||l}
 & r_2 = Y; \\
 & \mathsf{if}(r_2 == 1)\{ \\
 & \quad X = 1; \\
 & \quad r_3 = Z; \\
 & \quad \mathsf{if}(r_3 == 1) \\
 & \qquad X = 1; \\
r_1 = X; \quad\quad & \quad Z = 1; \\
\mathsf{if}(r_1 == 1) & \} \\
Y = 1; & \mathsf{else}\{ \\
 & \quad Z = 1; \\
 & \quad r_3 = 1; \\
 & \quad X = 1; \\
 & \}
\end{array}
\quad \rightsquigarrow \quad
\begin{array}{r||l}
 & d : X = 1; \\
 & e : r_2 = Y; \\
 & f : \mathsf{if}(r_2 == 1)\{ \\
 & g : \quad r_3 = Z; \\
 & \quad \mathsf{if}(r_3 == 1) \\
a : r_1 = X; & \qquad X = 1; \\
b : \mathsf{if}(r_1 == 1) & \quad Z = 1; \\
c : \quad Y = 1; & \} \\
 & \mathsf{else}\{ \\
 & \quad Z = 1; \\
 & \quad r_3 = 1; \\
 & \}
\end{array}
$$

Now we can have an interleaving d, a, b, c, e, f which results in $r_1 == r_2 == 1, r_3 == 0$.

C PROOFS OF DRF THEOREMS

First we prove the following lemma.

Lemma 5. *Given a program $\mathbb{P}$, suppose all its* RC11*-consistent executions are* RLX*-race-free. Let G be an event structure such that $G_{\mathsf{init}} \to_{\mathbb{P},\textsc{weakestmo}}{}^{*} G$. Then, $G.\mathsf{jf} \subseteq G.\mathsf{hb}$ holds.*

Proof. We show $G.\mathsf{jf} \subseteq G.\mathsf{hb}$ holds by induction on the construction of G. It holds trivially for $G = G_{\mathsf{init}}$ because $G_{\mathsf{init}}.\mathsf{jf} = \emptyset$.

For the inductive case, we know that $G_{\mathsf{init}} \to_{\mathbb{P},\textsc{weakestmo}}{}^{*} G \to_{\mathbb{P},\textsc{weakestmo}} G'$ and $G.\mathsf{jf} \subseteq G.\mathsf{hb}$, and have to show that $G'.\mathsf{jf} \subseteq G'.\mathsf{hb}$. We do case analysis on the step $G \to_{\mathbb{P},\textsc{weakestmo}} G'$; let e be the event appended to G to construct G'.

Case $e \notin \mathcal{R}$. In this case, $G'.\mathsf{jf} = G.\mathsf{jf}$ and $G.\mathsf{hb} \subseteq G'.\mathsf{hb}$. Hence $G'.\mathsf{jf} \subseteq G'.\mathsf{hb}$ holds.

Case $e \in \mathcal{R}$. In this case, there exists a write $w \in G.\mathsf{E}$ such that $G'.\mathsf{jf} = G.\mathsf{jf} \uplus \{(w, e)\}$. We consider the following cases for $G.\mathsf{jf}(w, e)$:

Subcase $(w, e) \in G'.\mathsf{hb}$. In this case, $G'.\mathsf{jf} \subseteq G'.\mathsf{hb}$ holds.

Subcase $(e, w) \in G'.\mathsf{hb}$. This case is not possible as it violates (COH′) in G'.

Subcase $(w, e) \notin G'.\mathsf{hb}^{=}$. In this case, $(w, e) \in G'.\mathsf{Race}(\textsc{rlx})$.

We take A to be the $G'.\mathsf{hb}$-prefixes of e and w. From (CFJ), it follows that A is conflict-free.

Let G'' be the restriction of G' to A. By construction, G'' is conflcit-free WEAKESTMO consistent event structure which is an RC11 execution and $(w, e) \in G''.\mathsf{Race}(\textsc{rlx})$. This contradicts the antecedent, and hence the statement holds. ◻

Lemma 6. *Given a program $\mathbb{P}$, suppose all its* RC11*-consistent executions are* RLX*-race-free. Then $\mathsf{X}.\mathsf{rf} \subseteq G.\mathsf{jf}$ holds where X is an execution extracted from* WEAKESTMO *event structure G, that is, $G_{\mathsf{init}} \to_{\mathbb{P},\textsc{weakestmo}}{}^{*} G$ and $\mathsf{X} \in \mathsf{ex}_{\textsc{weakestmo}}(G)$.*

Proof. Assume $(w_1, r) \in \mathsf{X}.\mathsf{rf} \setminus G.\mathsf{jf}$.

In this case there exists w_2 such that $G.\mathsf{ew}(w_1, w_2) \land (w_2, r) \in G.\mathsf{jf}$.

From Lemma 5 we know $(w_2, r) \in G.\mathsf{hb}$.

From the definition of X we know $w_2 \in \mathsf{X}.\mathsf{E}$.

It contradicts that $w_1 \in \mathsf{X}.\mathsf{E}$ and hence the statement holds. ◻

Lemma 7. *Given a program $\mathbb{P}$, suppose all its* RC11*-consistent executions are* RLX*-race-free. Then X has no $(\mathsf{X}.\mathsf{po} \cup \mathsf{X}.\mathsf{rf})$ cycle where X is an execution extracted from* WEAKESTMO *event structure G, that is, $G_{\mathsf{init}} \to_{\mathbb{P},\textsc{weakestmo}}{}^{*} G$ and $\mathsf{X} \in \mathsf{ex}_{\textsc{weakestmo}}(G)$.*

Proof. From Lemmas 5 and 6 we know $(\mathsf{X}.\mathsf{po} \cup \mathsf{X}.\mathsf{rf}) \subseteq (G.\mathsf{po} \cup G.\mathsf{jf}) \subseteq G.\mathsf{hb}$. Hence X has no $(\mathsf{X}.\mathsf{po} \cup \mathsf{X}.\mathsf{rf})$ cycle. ◻

Now we restate and prove the DRF-RLX theorem.

Theorem 2 (DRF-RLX) Given a program $\mathbb{P}$, suppose its RC11-consistent executions are RLX-race-free. Then, $\mathsf{Behavior}_{\textsc{weakestmo}}(\mathbb{P}) = \mathsf{Behavior}_{\mathsf{RC11}}(\mathbb{P})$.

Proof. Consider an extracted execution X from WEAKESTMO event structure G,

that is, $G_{\mathsf{init}} \to_{\mathbb{P},\textsc{weakestmo}}{}^{*} G$ and $\mathsf{X} \in \mathsf{ex}_{\textsc{weakestmo}}(G)$.

From Lemma 7 we know X has no $(\mathsf{X}.\mathsf{po} \cup \mathsf{X}.\mathsf{rf})$ cycle.

Hence X is an RC11 execution where $\mathsf{X}.\mathsf{rf} = \mathsf{X}.\mathsf{jf}$ and as a result, $\mathsf{Behavior}_{\textsc{weakestmo}}(\mathbb{P}) = \mathsf{Behavior}_{\mathsf{RC11}}(\mathbb{P})$ holds. ◻

D WEAKESTMO-LLVM CONSTRUCTION RULES

$$\dfrac{\begin{array}{c} A \subseteq G.\mathsf{E}_{e.\mathsf{tid}} \qquad \mathsf{dom}([\mathsf{E}_{e.\mathsf{tid}}]\,;\mathsf{po}\,;[A]) \subseteq A \qquad \mathit{labels}(\mathsf{sequence}_{\mathsf{po}}(A)) \cdot e.\mathsf{lab} \in \mathbb{P}(e.\mathsf{tid}) \\ \mathsf{E}' = \mathsf{E} \uplus \{e\} \qquad \mathsf{po}' = \mathsf{po} \cup (A \times \{e\}) \qquad \mathsf{isCons}_M(\langle \mathsf{E}', \mathsf{po}', \mathsf{jf}', \mathsf{ew}', \mathsf{mo}' \rangle) \qquad CF = (\mathsf{E}_{e.\mathsf{tid}} \setminus A) \\ \textbf{if } e \in \mathcal{R} \textbf{ then } \exists w \in \mathsf{E} \cap \mathcal{W}.\ \mathsf{jf}' = \mathsf{jf} \cup \{(w,e)\} \wedge w.\mathsf{loc} = e.\mathsf{loc} \wedge \\ ((w,e) \in G'.\mathsf{Race}(\textsc{na}) \wedge e.\mathsf{rval} = \mathbf{u} \vee w.\mathsf{wval} = e.\mathsf{rval}) \\ \textbf{else } \mathsf{jf}' = \mathsf{jf} \\ EW \subseteq \{w \in \mathcal{W} \cap CF \mid w.\mathsf{loc} = e.\mathsf{loc} \wedge w.\mathsf{wval} = e.\mathsf{wval}\} \qquad \mathsf{ew}' = \mathsf{ew} \cup (W \times \{e\})^{=} \\ W \subseteq AW = \{w \in \mathcal{W} \cap \mathsf{E} \setminus CF \mid w.\mathsf{loc} = e.\mathsf{loc} \wedge e \in \mathcal{W}\} \quad \mathsf{mo}' = \mathsf{mo} \cup W \times \{e\} \cup \{e\} \times (AW \setminus W) \end{array}}{\langle \mathsf{E}, \mathsf{po}, \mathsf{jf}, \mathsf{ew}, \mathsf{mo} \rangle \rightarrow_{\mathbb{P},M} \langle \mathsf{E}', \mathsf{po}', \mathsf{jf}', \mathsf{ew}', \mathsf{mo}' \rangle}$$

Fig. 33. WEAKESTMO-LLVM event structure construction rules where $G' = \langle \mathsf{E}', \mathsf{po}', \mathsf{jf}', \mathsf{ew}', \mathsf{mo}' \rangle$. The LLVM specific change is in green.

E MONOTONICITY OF WEAKESTMO

The WEAKEN transformation is as follows:

- $\tau \cdot \mathsf{Ld}_o(x, v) \cdot \tau' \xrightarrow{\text{WEAKEN}} \tau \cdot \mathsf{Ld}_{o'}(x, v) \cdot \tau'$ where $o' \sqsubseteq o$
- $\tau \cdot \mathsf{St}_o(x, v) \cdot \tau' \xrightarrow{\text{WEAKEN}} \tau \cdot \mathsf{St}_{o'}(x, v) \cdot \tau'$ where $o' \sqsubseteq o$
- $\tau \cdot \mathsf{U}_o(x, v, v') \cdot \tau' \xrightarrow{\text{WEAKEN}} \tau \cdot \mathsf{U}_{o'}(x, v, v') \cdot \tau'$ where $o' \sqsubseteq o$
- $\tau \cdot \mathcal{F}_o \cdot \tau' \xrightarrow{\text{WEAKEN}} \tau \cdot \mathcal{F}_{o'} \cdot \tau'$ where $o' \sqsubseteq o$
- $\tau \cdot \mathcal{F}_o \cdot \tau' \xrightarrow{\text{WEAKEN}} \tau \cdot \tau'$ where $o' \sqsubseteq o$

We prove that the WEAKESTMO is a monotonic memory model.

Theorem 9. *Given a program $\mathbb{P}_{\mathsf{src}}$ if we WEAKEN a program $\mathbb{P}_{\mathsf{src}}$ to $\mathbb{P}_{\mathsf{tgt}}$ then*
(1) for each consistent event srtucture of $\mathbb{P}_{\mathsf{src}}$ there exists a consistent event structure of $\mathbb{P}_{\mathsf{tgt}}$.
(2) for each consistent execution extracted from a consistent event srtucture of $\mathbb{P}_{\mathsf{src}}$ there exists a consistent execution extracted from a consistent event structure of $\mathbb{P}_{\mathsf{tgt}}$.

Formal statement

$$
\begin{array}{l}
\forall \mathbb{P}_{\mathsf{src}}.\ \text{WEAKEN}(\mathbb{P}_{\mathsf{src}}, \mathbb{P}_{\mathsf{tgt}}) \implies \\
\quad \forall G_{\mathsf{src}}.\ G_{\mathsf{init}} \longrightarrow_{\mathbb{P}_{\mathsf{src}},\text{WEAKESTMO}}{}^{*}\ G_{\mathsf{src}}.\ \exists G_{\mathsf{tgt}}.\ G_{\mathsf{init}} \longrightarrow_{\mathbb{P}_{\mathsf{tgt}},\text{WEAKESTMO}}{}^{*}\ G_{\mathsf{tgt}} \land \\
\qquad \forall \mathsf{X}_s \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{src}}).\ \exists \mathsf{X}_t \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{tgt}}).\ \mathsf{Behavior}(\mathsf{X}_t) = \mathsf{Behavior}(\mathsf{X}_s)
\end{array}
$$

Proof. (1) Given a target event structure $G_{\mathsf{init}} \longrightarrow_{\mathbb{P}_{\mathsf{src}},\text{WEAKESTMO}}{}^{*}\ G_{\mathsf{src}}$, we follow the construction steps of G_{src} and construct G_{tgt}. In this construction, we can follow the write steps similar to that of G_{tgt}. We can also follow the G_{src} fence step unless the fence is deleted. Hence we can append the reads with same labels by justifying from same writes compared to that of G_{src}. Thus, $G_{\mathsf{tgt}}.\mathsf{E} \subseteq G_{\mathsf{src}}.\mathsf{E}$, $G_{\mathsf{tgt}}.\mathcal{RW}_{o'} \equiv G_{\mathsf{tgt}}.\mathcal{RW}_{o}$, $G_{\mathsf{tgt}}.\mathsf{po} \subseteq G_{\mathsf{src}}.\mathsf{po}$, $G_{\mathsf{tgt}}.\mathsf{jf} = G_{\mathsf{src}}.\mathsf{jf}$, and $G_{\mathsf{tgt}}.\mathsf{ew} = G_{\mathsf{src}}.\mathsf{ew}$. While constructing G_{tgt} from G_{src}, essentially we remove po edges to/from fences along with certain sw edges due to the removal of fences or replacing the Rel or Acq events with events with weaker or same memory order. As a result, we in turn remove certain hb relations and the relations between the SC accesses.

As a result, the G_{tgt} is less restrictive than G_{src} in terms of the relations involved in the WEAKEST or WEAKESTMO consistency conditions and G_{tgt} remains consistent.

(2) For each execution $\mathsf{X}_s \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{src}})$, we find an execution X_t such that $\mathsf{X}_t.\mathsf{E} \subseteq \mathsf{X}_s.\mathsf{E}$, $\mathsf{X}_t.\mathcal{RW}_{o'} \equiv \mathsf{X}_s.\mathcal{RW}_{o}$, $\mathsf{X}_t.\mathsf{po} \subseteq \mathsf{X}_s.\mathsf{po}$, $\mathsf{X}_t.\mathsf{rf} = \mathsf{X}_s.\mathsf{rf}$, $\mathsf{X}_t.\mathsf{mo} = \mathsf{X}_s.\mathsf{mo}$.

Similiar to the event structures, the X_t is less restrictive than X_s in terms of the relations involved in the execution consistency conditions. Hence X_t remains consistent and $\mathsf{X}_t \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{tgt}})$ holds. Moreover, in this case $\mathsf{Behavior}(\mathsf{X}_s) = \mathsf{Behavior}(\mathsf{X}_t)$ holds folllowing the definitions of X_s and X_t. ◻

Remark 3. Consider we append a read r to consistent event structure G by justifying from a write $w \in G.\mathcal{W}$ from $(G'.\mathsf{hb} \cup G'.\mathsf{jf})$-prefix and create G' such that G' is consistent when $\mathsf{existsW}(G', w, r)$ holds where

$$\mathsf{existsW}(G', w, r) \triangleq (w, r) \in (G'.\mathsf{jf}^?; G'.\mathsf{hb}^? \setminus G'.\mathsf{ecf}) \land \nexists w'.\ \mathsf{existsW}(G', w', r) \land G'.\mathsf{mo}(w, w')$$

Note that there exists some write $w \in G.\mathcal{W}$ such that $\mathsf{existsW}(G, w, r)$ holds as all locations are initialized.

F PROOFS OF CORRECTNESS OF REORDERINGS

We start with definitions and a lemma on hb in the WEAKESTMO model.

We first define unique predecessor and unique successor.

Definition 8. $\mathrm{Upred}(R, a, b) \triangleq R(a, b) \land \forall c.\ G.R(c, b) \implies c = a$

Definition 9. $\mathrm{Usucc}(R, a, b) \triangleq R(a, b) \land \forall c.\ G.R(a, c) \implies c = b$.

We derive the following lemma.

Lemma 8. *if* $\mathrm{Upred}(R, b, a)$ *and* $\mathrm{Usucc}(R, b, a)$ *holds then* $(R \setminus \{(b, a)\} \cup \{(a, b)\})^+ \subseteq R^+ \setminus \{(b, a)\} \cup \{(a, b)\}$ *also holds.*

Proof. We assume $\mathrm{Upred}(R, b, a)$ and $\mathrm{Usucc}(R, b, a)$ holds.
Now we show $(R \setminus \{(b, a)\} \cup \{(a, b)\})^+ \subseteq R^+ \setminus \{(b, a)\} \cup \{(a, b)\}$.
We prove by induction on transitive closure.
Base Case: $R \setminus \{(b, a)\} \cup \{(a, b)\} \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.
The base case holds trivially by monotonicity.

The induction step:
$(R \setminus \{(b, a)\} \cup \{(a, b)\}) \circ (R^+ \setminus \{(b, a)\} \cup \{(a, b)\}) \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.
To prove the above mentioned induction, we consider following cases

case 1. $(R \setminus \{(b, a)\}) \circ (R^+ \setminus \{(b, a)\}) \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.
It is sufficient to show:
$(R \setminus \{(b, a)\}) \circ (R^+ \setminus \{(b, a)\}) \subseteq R^+ \setminus \{(b, a)\}$
Therefore it is sufficient to show,
$(R \setminus \{(b, a)\}) \circ (R^+ \setminus \{(b, a)\}) \subseteq R^+ \land (b, a) \notin (R \setminus \{(b, a)\}) \circ (R^+ \setminus \{(b, a)\})$.
Now
(i) By monotonicity we know that $(R \setminus \{(b, a)\}) \circ (R^+ \setminus \{(b, a)\}) \subseteq R^+$.
therefore it is sufficient to show
(ii) $(b, a) \notin (R \setminus \{(b, a)\}) \circ (R^+ \setminus \{(a, b)\})$.
Assume $(b, a) \in (R \setminus \{(b, a)\}) \circ (R^+ \setminus \{(b, a)\})$.
By unfolding the definition of $\circ$, it is sufficient to show
$\nexists c.\ (b, c) \in (R \setminus \{(b, a)\}) \land (c, a) \in (R^+ \setminus \{(b, a)\})$.
Assume $\exists c. (b, c) \in R \setminus \{(b, a)\}$.
Therefore $(b, c) \in R \land c \neq a \land (c, a) \in R^+ \land c \neq b$.
From $\mathrm{Usucc}(R, b, a)$ we know $c = a$ which is a contradiction.
Hence $\nexists c.\ (b, c) \in (R \setminus \{(b, a)\})$.

case 2. $(R \setminus \{(b, a)\}) \circ \{(a, b)\} \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.
We know $\mathrm{Upred}(R, a, b)$ holds and hence $\nexists a, b, c.\ R(b, a) \land R(c, a) \land b \neq c$.
Hence, $R \setminus \{(b, a)\} \circ \{(a, b)\} = \emptyset$.
As a result, $R \setminus \{(b, a)\} \circ \{(a, b)\} \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.

case 3. $\{(a, b)\} \circ (R^+ \setminus \{(b, a)\} \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.
We know $\{(a, b)\} \circ R \setminus \{(b, a)\} = \emptyset$ because $\mathrm{Usucc}(R, a, b)$ holds, that is, $\nexists a, b, c.\ R(a, b) \land R(a, c) \land b \neq c$.
As a result, $\{(a, b)\} \circ R \setminus \{(b, a)\} \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.

case 4. $\{(a, b)\} \circ \{(a, b)\} \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.
$\{(a, b)\} \circ \{(a, b)\} = \emptyset$ and hence $\{(a, b)\} \circ \{(a, b)\} \subseteq (R^+ \setminus \{(b, a)\} \cup \{(a, b)\})$.

□

Now we relate the happens-before relations between the source and target executions. The safe reorderings from Table 1 as follows:

reord($\mathbb{P}_{\text{src}}, \mathbb{P}_{\text{tgt}}$) such that

$\mathbb{P}_{\text{tgt}}(i) \subseteq \mathbb{P}_{\text{src}}(i) \cup \{\tau{\cdot}\beta{\cdot}\tau' \mid \tau{\cdot}\alpha{\cdot}\tau' \in \mathbb{P}_{\text{src}}(i)\} \land \forall j \neq i.\ \mathbb{P}_{\text{tgt}}(j) = \mathbb{P}_{\text{src}}(j)$

where $\alpha = a{\cdot}b, \beta = b{\cdot}a,$ and a, b are labels of shared memory accesses or fences..

Lemma 9. *Suppose*

(1) reord($\mathbb{P}_{\text{src}}, \mathbb{P}_{\text{tgt}}$) *where the reordering is* $a; b \rightsquigarrow b; a$ *and*

(2) $\mathsf{X}_s \in \mathsf{ex}_{\textit{WEAKESTMO}}(G_{\text{src}})$ *where* $G_{\text{init}} \rightarrow_{\mathbb{P}_{\text{src}},\textit{WEAKESTMO}}{}^{*} G_{\text{src}}$ *and*

(3) $\mathsf{X}_t \in \mathsf{ex}_{\textit{WEAKESTMO}}(G_{\text{tgt}})$ *where* $G_{\text{init}} \rightarrow_{\mathbb{P}_{\text{tgt}},\textit{WEAKESTMO}}{}^{*} G_{\text{tgt}}$.

Then $\mathsf{X}_s.\mathsf{hb}_{\mathsf{C11}} \subseteq (\mathsf{X}_t.\mathsf{hb}_{\mathsf{C11}} \setminus \{(b,a)\} \cup \{(a,b)\})$.

Proof. We know $\mathsf{X}_s.\mathsf{po} = \mathsf{X}_t.\mathsf{po} \setminus \{(b,a)\} \cup \{(a,b)\}$. Let $R = (\mathsf{X}_t.\mathsf{po} \cup R')$ where R' is some other relation independent of $\mathsf{X}_t.\mathsf{po}$. Hence from Lemma 8,

$$(R \setminus \{(b,a)\} \cup \{(a,b)\})^+ \subseteq (R^+ \setminus \{(b,a)\} \cup \{(a,b)\})$$

$$\implies ((\mathsf{X}_t.\mathsf{po} \cup R') \setminus \{(b,a)\} \cup \{(a,b)\})^+ \subseteq ((\mathsf{X}_t.\mathsf{po} \cup R')^+ \setminus \{(b,a)\} \cup \{(a,b)\})$$

$$\implies ((\mathsf{X}_t.\mathsf{po} \setminus \{(b,a)\} \cup \{(a,b)\}) \cup R')^+ \subseteq ((\mathsf{X}_t.\mathsf{po} \cup R')^+ \setminus \{(b,a)\} \cup \{(a,b)\})$$

$$\implies (\mathsf{X}_s.\mathsf{po} \cup R')^+ \subseteq ((\mathsf{X}_t.\mathsf{po} \cup R')^+ \setminus \{(b,a)\} \cup \{(a,b)\})$$

$$\implies (\mathsf{imm}(\mathsf{X}_s.\mathsf{po}) \cup R')^+ \subseteq ((\mathsf{imm}(\mathsf{X}_t.\mathsf{po}) \cup R')^+ \setminus \{(b,a)\} \cup \{(a,b)\})$$

since $(\mathsf{X}_s.\mathsf{po} \cup R')^+ = (\mathsf{imm}(\mathsf{X}_s.\mathsf{po}) \cup R')^+$ and $(\mathsf{X}_t.\mathsf{po} \cup R')^+ = (\mathsf{imm}(\mathsf{X}_t.\mathsf{po}) \cup R')^+$,
substituting $R' = \mathsf{X}_s.\mathsf{sw}_{\mathsf{C11}} = \mathsf{X}_t.\mathsf{sw}_{\mathsf{C11}}$ we get

$$(\mathsf{imm}(\mathsf{X}_s.\mathsf{po}) \cup \mathsf{X}_s.\mathsf{sw}_{\mathsf{C11}})^+ \subseteq ((\mathsf{X}_t.\mathsf{po} \cup \mathsf{X}_t.\mathsf{sw}_{\mathsf{C11}})^+ \setminus \{(b,a)\} \cup \{(a,b)\})$$

It implies $\mathsf{X}_s.\mathsf{hb}_{\mathsf{C11}} \subseteq (\mathsf{X}_t.\mathsf{hb}_{\mathsf{C11}} \setminus \{(b,a)\} \cup \{(a,b)\})$
as $\mathsf{X}_s.\mathsf{hb}_{\mathsf{C11}} = (\mathsf{imm}(\mathsf{X}_s.\mathsf{po}) \cup \mathsf{X}_s.\mathsf{sw}_{\mathsf{C11}})^+$ and $\mathsf{X}_t.\mathsf{hb}_{\mathsf{C11}} = (\mathsf{imm}(\mathsf{X}_t.\mathsf{po}) \cup \mathsf{X}_t.\mathsf{sw}_{\mathsf{C11}})^+$. ◻

F.1 Reordering Theorem

We restate the definition of compilation correctness and the safe reordering theorem.

Definition 7. A transformation of program $\mathbb{P}_{\text{src}}$ in memory model M_{src} to program $\mathbb{P}_{\text{tgt}}$ in model M_{tgt} is *correct* if it does not introduce new behaviors: i.e., $\mathsf{Behavior}_{M_{\text{tgt}}}(\mathbb{P}_{\text{tgt}}) \subseteq \mathsf{Behavior}_{M_{\text{src}}}(\mathbb{P}_{\text{src}})$.

Theorem 6. *The safe reorderings in Table 1 are correct in both* WEAKESTMO *models.*

The formal statement is as follows:

$$\begin{array}{l}
\forall \mathbb{P}_{\text{src}}.\ \mathsf{reord}(\mathbb{P}_{\text{src}}, \mathbb{P}_{\text{tgt}}) \implies \\
\quad \forall G_{\text{tgt}}.\ G_{\text{init}} \rightarrow_{\mathbb{P}_{\text{tgt}},\textsc{weakestmo}}{}^{*} G_{\text{tgt}}.\ \exists G_{\text{src}}.\ G_{\text{init}} \rightarrow_{\mathbb{P}_{\text{src}},\textsc{weakestmo}}{}^{*} G_{\text{src}} \land \\
\quad\quad \forall \mathsf{X}_t \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\text{tgt}}).\ \exists \mathsf{X}_s \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\text{src}}).\ \mathsf{Behavior}(\mathsf{X}_t) = \mathsf{Behavior}(\mathsf{X}_s) \\
\quad \land \mathsf{X}_t.\mathsf{Race} \cap \mathcal{E}_{\text{NA}} \neq \emptyset \implies \mathsf{X}_s.\mathsf{Race} \cap \mathcal{E}_{\text{NA}} \neq \emptyset
\end{array}$$

To prove the theorem, given an extracted consistent target execution $\mathsf{X}_t \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\text{tgt}})$ from a consistent target event structure G_{tgt}, we construct a consistent source execution X_s from X_t. Then we ensure that the behavior of the X_s and X_t are same and if X_t has undefined behavior due to data race then X_s also has undefined behavior due to data race. Finally, we show that the $\mathsf{X}_s \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\text{src}})$ where G_{src} is a WEAKESTMO consistent source event structure.

Proof. In this proof we follow the above mentioned steps as follows.

Source Execution Consistency. From target execution X_t we get source execution X_s by reordering the respective events. Thus if $\mathsf{imm}(\mathsf{X}_t.\mathsf{po})(b, a)$ then $\mathsf{imm}(\mathsf{X}_t.\mathsf{po})(a, b)$ holds. We know, following the Lemma 9, $\mathsf{X}_s.\mathsf{hb} \subseteq \mathsf{X}_t \setminus \{(b, a)\} \cup \{(a, b)\}$, that is, X_s is more relaxed than X_t. We also know that X_t is consistent. Hence the execution X_s is consistent.

Same Behavior. The behaviors of X_s and X_t are same. The reordering does not introduce any new mo relation in X_s and thus $\mathsf{X}_t.\mathsf{mo} = \mathsf{X}_s.\mathsf{mo}$. Hence the behaviors of X_s and X_t are same.

Race Preservation.

following the Lemma 9, $\mathsf{X}_s.\mathsf{hb} \subseteq \mathsf{X}_t.\mathsf{hb} \setminus \{(b, a)\} \cup \{(a, b)\}$. Hence if X_t is racy, then X_s is also racy. As a result, if the target execution has undefined behavior due to a data race, so does the source execution.

Source Event Structure Construction and Execution Extraction

It is left to show that we can construct a source event structure $G_{\mathsf{init}} \rightarrow_{\mathbb{P}_{\mathsf{src}},\textsc{weakestmo}}{}^{*}\, G_{\mathsf{src}}$ such that execution X_s is an extracted execution from G_{src}, that is, $\mathsf{X}_s \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\mathsf{src}})$.

If $(\mathsf{X}_s.\mathsf{po} \cup \mathsf{X}_s.\mathsf{rf})^+$ is acyclic, then we follow the $(\mathsf{X}_s.\mathsf{po} \cup \mathsf{X}_s.\mathsf{rf})^+$ path to construct the source event structure and in this case $G_{\mathsf{src}} = \mathsf{X}_s$. From the definitions we know that WEAKESTMO constraints are weaker than the execution constraints. Hence G_{src} is consistent as X_s is consistent. As a result, $\mathsf{X}_s \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\mathsf{src}})$.

However, if X_s has $(\mathsf{X}_s.\mathsf{po} \cup \mathsf{X}_s.\mathsf{rf})^+$ cycle(s), then we construct G_{src} and extract X_s from G_{src}.

Source Event Structure Construction. To construct G_{src}, we follow the construction steps of G_{tgt}. For each target construction step that adds event e to G_{tgt} to get G'_{tgt}, we perform one or more corresponding steps going from G_{src} to G'_{src}. We do a case analysis on the event e of the target event structure. For the reordered events the construction is as follows:

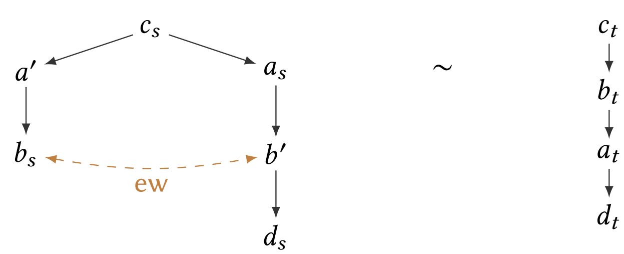


Fig. 34. $\{(c_s, c_t), (b_s, b_t), (a_s, a_t), (b', b_t), (d_s, d_t)\} \subseteq \mathbb{M}$.

We define $\mathsf{pc} : \mathsf{N} \rightarrow \mathsf{E}$; a function that maps a thread identifier to an event in the respective thread in the execution.

We use pc to keep track of the X_s in G_{src}.

We define $\mathbb{M}$ relation which pairs a G_{src} and G_{tgt} event, that is,

$$\mathbb{M} \triangleq \{(s, t) \mid s \in G_{\mathsf{src}}.\mathsf{E} \wedge t \in G_{\mathsf{tgt}}.\mathsf{E} \wedge s.\mathsf{lab} = t.\mathsf{lab} \wedge s.\mathsf{tid} = t.\mathsf{tid}\}$$

Let $A \subseteq G_{\mathsf{tgt}}.\mathsf{E}$, $B \subseteq G_{\mathsf{tgt}}.\mathsf{E}$ denote the pair of sets of events which are created for the reordered access pairs.

We call $A \cup B$ as *reordered* events and $G_{\mathsf{tgt}}.\mathsf{E} \setminus (A \cup B)$ as *non-reordered* events.

Also let $C \subseteq G_{\mathsf{tgt}}.\mathsf{E} \setminus (A \cup B)$ be the immediate $G_{\mathsf{tgt}}.\mathsf{po}$-predecessors of the B events.

We say $G_{\mathsf{src}} \sim G_{\mathsf{tgt}}$ holds iff

(1) G_{src}, G_{tgt} are consistent.

(2) there exists $\mathbb{M}$ such that G_{src} and G_{tgt} preserves invariant which is a conjunction of following clauses.

(a) The non-reordered events in the target event structures are mapped to some non-reordered events in the source event structure.

$$\forall c_t \in G_{\text{tgt}}.\mathsf{E} \setminus (A \cup B).\ \exists c_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t)$$

(b) If b_t is po-successor of some event c_t in the target event structure then there exists a', b_s, c_s events in the source event structure such that $\mathbb{M}(b_s, b_t)$, $\mathbb{M}(c_s, c_t)$ hold. In addition, memory location and memory order of a' and a_t match.

$$\begin{array}{l}\forall c_t \in G_{\text{tgt}}.\mathsf{E} \setminus (A \cup B), a_t \in A, b_t \in B \wedge G_{\text{tgt}}.\mathsf{po}(c_t, b_t) \implies \\ \exists c_s, a_s, b_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \\ \wedge(\exists a' \in G_{\text{src}}.\mathsf{E}.\ a_s.\mathsf{loc} = a'.\mathsf{loc} \wedge a_s.\mathsf{ord} = a'.\mathsf{ord} \wedge G_{\text{src}}.\mathsf{po}(c_s, a') \\ \wedge \mathsf{imm}(G_{\text{src}}.\mathsf{po})(a', b_s))\end{array}$$

(c) If a_t is po-successor of some event c_t in the target event structure then there exists a_s, c_s events in the source event structure such that $\mathbb{M}(a_s, a_t)$ and $\mathbb{M}(c_s, c_t)$ hold.

$$\begin{array}{l}\forall c_t \in G_{\text{tgt}}.\mathsf{E} \setminus (A \cup B), a_t \in A. \wedge G_{\text{tgt}}.\mathsf{po}(c_t, a_t) \implies \\ \exists c_s, a_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(a_s, a_t) \wedge G_{\text{src}}.\mathsf{po}(c_s, a_s)\end{array}$$

(d) If $a_t \in A$ is immediate-po successor of $b_t \in B$ in the target event structure then there exist $a_s, a', b_s, b', c_s, c_t$ such that
 (i) $\{(c_s, c_t), (b_s, b_t), (a_s, a_t)\} \subseteq \mathbb{M}$ holds.
 (ii) c_s and c_t are non-reordered events such that if c_t is immediate-po-predecessor of b_t then c_s is immediate-po predecessor of a_s.
 (iii) a' and a are in immediate-conflict relation.
 (iv) b_s and b' are immediate-po successors of a' and a_s respectively.
 (v) b' and b_s are equal-writes.

$$\begin{array}{l}\forall a_t \in A, b_t \in B.\ \mathsf{imm}(G_{\text{tgt}}.\mathsf{po})(b_t, a_t) \implies \\ (\exists c_t \in G_{\text{tgt}}.\mathsf{E} \setminus (A \cup B), a', a_s, b_s, c_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \\ \wedge \mathsf{imm}(G_{\text{tgt}}.\mathsf{po})(c_t, b_t) \wedge \mathsf{imm}(G_{\text{src}}.\mathsf{po})(c_s, a_s) \wedge \mathsf{imm}(G_{\text{src}}.\mathsf{po})(a_s, b') \\ \wedge \mathsf{imm}(G_{\text{src}}.\mathsf{cf})(a_s, a') \wedge \mathsf{imm}(G_{\text{src}}.\mathsf{po})(a', b_s) \\ \wedge b_s.\mathsf{loc} = b'.\mathsf{loc} \wedge b_s.\mathsf{ord} = b'.\mathsf{ord} \wedge G_{\text{src}}.\mathsf{ew}(b_s, b'))\end{array}$$

(e) If non-reordered event c_t is po-successor of b_t in the target event structure then there exists c_s in source event structure which maps to c_t and c_s is po-successor of b' or b_s where b' and b_s are equal-writes.

$$\begin{array}{l}\forall c_t \in G_{\text{tgt}}.\mathsf{E} \setminus (A \cup B), b_t \in B.\ G_{\text{tgt}}.\mathsf{po}(b_t, c_t) \implies \\ \exists b_s, b', c_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(b_s, b_t) \wedge \mathbb{M}(b', b_t) \\ \wedge G_{\text{src}}.\mathsf{ew}(b_s, b') \wedge (G_{\text{src}}.\mathsf{po}(b_s, c_s) \vee G_{\text{src}}.\mathsf{po}(b', c_s))\end{array}$$

(f) If $a_t \in A$ is immediate-po successor of $b_t \in B$ in the target event structure then there is no po relation between b_s and a_s in source event structure where a_s maps to a_t and b_s maps to b_t.

$$\begin{array}{l}\forall a_t \in A, b_t \in B.\ G_{\text{tgt}}.\mathsf{po}(b_t, a_t) \implies \\ \exists a_s, b_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge \neg G_{\text{src}}.\mathsf{po}(b_s, a_s)\end{array}$$

(g) For a pair of non-ordered events in the target event structure which are in po relation, there exists corresponding pair of events in the source event structure chich are in po relation.

$$\begin{array}{l}\forall c_t, c'_t \in G_{\mathrm{tgt}}.\mathsf{E} \setminus (A \cup B).\ G_{\mathrm{tgt}}.\mathsf{po}(c_t, c'_t) \implies \\ \exists c_s, c'_s \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(c'_s, c'_t) \wedge G_{\mathrm{src}}.\mathsf{po}(c_s, c'_s)\end{array}$$

(h) If a_t is justified from an event c_t in the target event structure then there exists corresponding a_s, c_s events in the source event structure such that a_s is justified from c_s.

$$\begin{array}{l}\forall c_t \in G_{\mathrm{tgt}}.\mathsf{E} \setminus (A \cup B), a_t \in A.\ G_{\mathrm{tgt}}.\mathsf{jf}(c_t, a_t) \implies \\ \exists c_s, a_s \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(c_s, c_t) \wedge G_{\mathrm{src}}.\mathsf{jf}(c_s, a_s)\end{array}$$

(i) If a_t justifies an event c_t in the target event structure then there exists corresponding a_s, c_s events in the source event structure such that a_s justifies c_s.

$$\begin{array}{l}\forall c_t \in G_{\mathrm{tgt}}.\mathsf{E} \setminus (A \cup B), a_t \in A.\ G_{\mathrm{tgt}}.\mathsf{jf}(a_t, c_t) \implies \\ \exists c_s, a_s \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(c_s, c_t) \wedge G_{\mathrm{src}}.\mathsf{jf}(a_s, c_s)\end{array}$$

(j) If b_t is justified from an event c_t in the target event structure then there exists corresponding b' and b_s, c_s events in the source event structure such that c_s justifies b_s, b', and b_s, b' are equal-writes.

$$\begin{array}{l}\forall c_t \in G_{\mathrm{tgt}}.\mathsf{E} \setminus (A \cup B), b_t \in B.\ G_{\mathrm{tgt}}.\mathsf{jf}(c_t, b_t) \implies \\ \exists b_s, c_s \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(b_s, b_t) \wedge \mathbb{M}(c_s, c_t) \wedge G_{\mathrm{src}}.\mathsf{jf}(c_s, b_s) \\ \wedge(\exists b' \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(b', b_t) \wedge G_{\mathrm{src}}.\mathsf{ew}(b_s, b') \implies G_{\mathrm{src}}.\mathsf{jf}(c_s, b'))\end{array}$$

(k) If b_t in the target event structure justifies c_t then either there exists b' corresponding to b_t such that b' justifies c_s where there is no b_s that maps to b_t or source event structure has b_s which is equal-writes to b' and justifies c_s.

$$\begin{array}{l}\forall c_t \in G_{\mathrm{tgt}}.\mathsf{E} \setminus (A \cup B), b_t \in B.\ G_{\mathrm{tgt}}.\mathsf{jf}(b_t, c_t) \implies \\ ((\exists b_s, c_s \in G_{\mathrm{src}}.\mathsf{E}.\ (\mathbb{M}(b_s, b_t) \wedge \nexists b' \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(b', b_t) \wedge G_{\mathrm{src}}.\mathsf{ew}(b_s, b')) \\ \qquad\qquad \implies G_{\mathrm{src}}.\mathsf{jf}(b_s, c_s)) \\ \vee(\exists b', b_s, c_s \in G_{\mathrm{src}}.\mathsf{E}.\ (\mathbb{M}(b_s, b_t) \wedge \mathbb{M}(b', b_t) \wedge \mathbb{M}(c_s, c_t) \wedge G_{\mathrm{src}}.\mathsf{ew}(b_s, b')) \\ \qquad\qquad \implies G_{\mathrm{src}}.\mathsf{jf}(b', c_s)))\end{array}$$

(l) If a pair of non-reordered events are in justified-from relation, then there exists corresponding pair of events in the source event structure in justified-from relation.

$$\begin{array}{l}\forall c_t, c'_t \in G_{\mathrm{tgt}}.\mathsf{E} \setminus (A \cup B).\ G_{\mathrm{tgt}}.\mathsf{jf}(c_t, c'_t) \implies \\ \exists c_s, c'_s \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(c'_s, c'_t) \wedge G_{\mathrm{src}}.\mathsf{jf}(c_s, c'_s)\end{array}$$

(m) If there is mo relation from a non-reordered event c_t to an ordered event a_t then there exists events c_s, a_s in mo relation in source event structure where non-reordered event c_s maps to c_t and ordered event a_s maps to a_t.

$$\begin{array}{l}\forall c_t \in G_{\mathrm{tgt}}.\mathsf{E} \setminus (A \cup B), a_t \in A, b_t \in B.\ G_{\mathrm{tgt}}.\mathsf{mo}(c_t, a_t) \implies \\ \exists c_s, a_s \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(a_s, a_t) \wedge G_{\mathrm{src}}.\mathsf{mo}(c_s, a_s)\end{array}$$

(n) If there is mo relation from an ordered event a_t to a non-reordered event c_t then there exists mo relation from event a_s to c_s in source event structure where ordered event a_s maps to a_t and non-reordered event c_s maps to c_t.

$$\begin{array}{l}\forall c_t \in G_{\mathrm{tgt}}.\mathsf{E} \setminus (A \cup B), a_t \in A.\ G_{\mathrm{tgt}}.\mathsf{mo}(a_t, c_t) \implies \\ \exists c_s, a_s \in G_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(a_s, a_t) \wedge G_{\mathrm{src}}.\mathsf{mo}(a_s, c_s)\end{array}$$

(o) If there is mo relation from a non-reordered event c_t to an ordered event b_t then there exists events c_s, b_s in mo relation in source event structure where non-reordered event c_s maps to c_t and ordered event b_s maps to b_t.

$$\forall c_t \in G_{\text{tgt}}.\mathsf{E} \setminus (A \cup B), b_t \in B.\ G_{\text{tgt}}.\text{mo}(c_t, b_t) \implies$$
$$\exists c_s, b_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(b_s, b_t) \wedge G_{\text{src}}.\text{mo}(c_s, b_s)$$

(p) If there is mo relation from an ordered event b_t to a non-reordered event c_t then there exists mo relation from event b_s to c_s in source event structure where ordered event b_s maps to b_t and non-reordered event c_s maps to c_t.

$$\forall c_t \in G_{\text{tgt}}.\mathsf{E} \setminus (A \cup B), b_t \in B.\ G_{\text{tgt}}.\text{mo}(b_t, c_t) \implies$$
$$\exists c_s, b_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(b_s, b_t) \wedge G_{\text{src}}.\text{mo}(b_s, c_s)$$

(q) If there is mo relation between a pair of non-reordered events c_t and c'_t in the target event structure then there exists mo relation from event c_s to c'_s in source event structure where c_s maps to c_t and c'_s maps to c'_t.

$$\forall c, c' \in G_{\text{tgt}}.\mathsf{E} \setminus (A \cup B).\ G_{\text{tgt}}.\text{mo}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(c'_s, c'_t) \wedge G_{\text{src}}.\text{mo}(c_s, c'_s)$$

(r) If an event is unmapped in the source event structure then there is no outgoing mo edge from that event.

$$\forall e_s \in G_{\text{src}}.\mathcal{W}.\ (\nexists e_t \in G_{\text{tgt}}.\mathsf{E}.\ \mathbb{M}(e_s, e_t)) \implies$$
$$\nexists e'_s \in G_{\text{src}}.\mathsf{E}.\ G_{\text{src}}.\text{mo}(e_s, e'_s)$$

(s) For each equal-writes pair of events in the target event structure, there exists equal-writes pairs in the source event structure.

$$\forall c_t, c'_t \in G_{\text{tgt}}.\mathsf{E}.\ G_{\text{tgt}}.\text{ew}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}(c'_s, c'_t) \wedge G_{\text{src}}.\text{ew}(c_s, c'_s)$$

(3) there exists pc such that
$\mathsf{X}_s.\mathsf{E} = \mathbb{S}$
$\mathsf{X}_s.\text{po} = G_{\text{src}}.\text{po} \cap (\mathbb{S} \times \mathbb{S})$
$\mathsf{X}_s.\text{rf} = G_{\text{src}}.\text{rf} \cap (\mathbb{S} \times \mathbb{S})$
$\mathsf{X}_s.\text{mo} = G_{\text{src}}.\text{mo} \cap (\mathbb{S} \times \mathbb{S})$
where $\mathbb{S}(G_{\text{src}}, \text{pc}) \triangleq \{e \mid e \in G_{\text{src}}.\mathsf{E} \wedge G_{\text{src}}.\text{po}^?(e, \text{pc}(e.\text{tid}))\}$.

To prove the simulation we show the followings.

$$G_{\text{src}} \sim G_{\text{tgt}} \wedge G_{\text{tgt}} \xrightarrow{\text{WEAKESTMO}} G'_{\text{tgt}} \implies \exists G'_{\text{src}}.\ G_{\text{src}} \xrightarrow{\text{WEAKESTMO}}_+ G'_{\text{src}} \wedge G'_{\text{src}} \sim G'_{\text{tgt}}$$

At each construction step, we extend G_{tgt} to G'_{tgt} by po-extending from an event $e_t \in G_{\text{tgt}}.\mathsf{E}$ with a new event $e'_t \in G'_{\text{tgt}}.\mathsf{E}$. We consider following cases:

Case $e'_t \in B'$ where $B' = B \uplus \{e'_t\}$:

In this case $A' = A$, and $G'_{\text{tgt}}.\mathsf{E} = G_{\text{tgt}}.\mathsf{E} \uplus \{e'_t\}$.

We also append corresponding event(s) in G_{src} and construct G'_{src}.

(1) Condition to show: G'_{src} *is consistent.*

The construction has two steps: $G_{\text{src}} \to G''_{\text{src}} \to G'_{\text{src}}$. In G''_{src} we introduce a' and in G'_{src} we introduce e'_s.

case. event e_s has an immediate po successor a'' such that $a.\text{loc} = a''.\text{loc}$ and $a.\text{ord} = a''.\text{ord}$. In this case $a' = a''$ and $G''_{\text{src}} = G_{\text{src}}$.

otherwise.

We append an event a' in G_{src} and create G''_{src} such that

$$
\begin{aligned}
G''_{\text{src}}.\mathsf{E} =& G_{\text{src}}.\mathsf{E} \uplus \{a'\} \\
G''_{\text{src}}.\mathsf{po} =& (G_{\text{src}}.\mathsf{po} \uplus \{(e_s, a') \mid \mathbb{M}(e_s, e_t)\})^+ \\
G''_{\text{src}}.\mathsf{jf} =& G_{\text{src}}.\mathsf{jf} \\
& \uplus \{(w, a') \mid (w, a') \in (G''_{\text{src}}.\mathcal{W} \times G''_{\text{src}}.\mathcal{R}) \\
& \wedge \exists w' \in G'_{\text{tgt}}.\mathsf{E}.\ \mathbb{M}(w, w') \wedge G'_{\text{tgt}}.\mathsf{jf}(w', a)\} \\
& \uplus \{(w, a') \mid (w, a') \in (G''_{\text{src}}.\mathcal{W} \times G''_{\text{src}}.\mathcal{R}) \\
& \wedge \nexists w' \in G'_{\text{tgt}}.\mathsf{E}.\ \mathbb{M}(w, w') \wedge G'_{\text{tgt}}.\mathsf{jf}(w', a) \wedge \mathsf{existsW}(G''_{\text{src}}, w, a')\} \\
G''_{\text{src}}.\mathsf{mo} =& G_{\text{src}}.\mathsf{mo} \uplus \{(w, a') \mid (w, a') \in (G''_{\text{src}}.\mathcal{W} \times G''_{\text{src}}.\mathcal{W})\} \\
G''_{\text{src}}.\mathsf{ew} =& G_{\text{src}}.\mathsf{ew}
\end{aligned}
$$

Also in this case $\mathbb{M}'' = \mathbb{M}$.
Now we check whether G''_{src} is consistent.
We know that $G_{\text{tgt}} \sim G_{\text{src}}$. Hence G_{src} and G_{tgt} are consistent.
If $G''_{\text{src}} = G_{\text{src}}$ then G''_{src} is consistent as G_{src} is consistent.
Otherwise, from definition of G''_{src} and observation from Remark 3 we know that G''_{src} satisfies (CF), (CFJ), (VISJ), (ICF), (ICFJ).
There is no outgoing edge from a' and hence it does not result in any $(G''_{\text{src}}.\mathsf{hb}; G''_{\text{src}}.\mathsf{eco}^?)$ cycle. Hence G''_{src} satisfies (COH′).
As a result, G''_{src} remains consistent.
Next, we construct G'_{src} from G''_{src}.
case. There exists e'_s where $e'_s.\mathsf{lab} = e'_t.\mathsf{lab}$ and if $e'_s, e'_t \in \mathcal{R}$ then $G''_{\text{src}}.\mathsf{jf}(w_s, e'_s)$, $G''_{\text{src}}.\mathsf{jf}(w_t, e'_s)$, $\mathbb{M}''(w_s, w_t)$ hold.
In this case $G'_{\text{src}} = G''_{\text{src}}$ and $b_s = e'_s$.
Otherwise. We append such a e'_s and thus

$$
\begin{aligned}
G'_{\text{src}}.\mathsf{E} =& G''_{\text{src}}.\mathsf{E} \uplus \{e'_s \mid e'_s.\mathsf{lab} = e'_t.\mathsf{lab}\} \\
G'_{\text{src}}.\mathsf{po} =& (G''_{\text{src}}.\mathsf{po} \uplus \{(a', e'_s)\})^+ \\
G'_{\text{src}}.\mathsf{jf} =& G''_{\text{src}}.\mathsf{jf} \\
& \uplus \{(w_s, e'_s) \mid (w_s, e'_s) \in (G'_{\text{src}}.\mathcal{W} \times G'_{\text{src}}.\mathcal{R}) \wedge G'_{\text{tgt}}.\mathsf{jf}(w_t, e'_t) \wedge \mathbb{M}''(w_s, w_t)\} \\
G'_{\text{src}}.\mathsf{mo} =& G''_{\text{src}}.\mathsf{mo} \\
& \uplus \{(w_s, e'_s) \mid (w_s, e'_s) \in (G'_{\text{src}}.\mathcal{W} \times G'_{\text{src}}.\mathcal{W}) \\
& \wedge \mathbb{M}''(w_s, w_t) \wedge G'_{\text{tgt}}.\mathsf{mo}(w_t, e'_t)\} \\
& \uplus \{(e'_s, w_s) \mid (w_s, e'_s) \in (G'_{\text{src}}.\mathcal{W} \times G'_{\text{src}}.\mathcal{W}) \wedge \\
& \mathbb{M}''(w_s, w_t) \wedge G'_{\text{tgt}}.\mathsf{mo}(e'_t, w_t)\} \\
G'_{\text{src}}.\mathsf{ew} =& G''_{\text{src}}.\mathsf{ew} \uplus \{(w_s, e'_s), (e'_s, w_s) \mid (w_s, e'_s) \in (G'_{\text{src}}.\mathcal{W}_{\sqsubseteq\textsc{rlx}} \times G'_{\text{src}}.\mathcal{W}_{\sqsubseteq\textsc{rlx}}) \\
& \wedge \mathbb{M}''(w_s, w_t) \wedge G'_{\text{tgt}}.\mathsf{ew}(w_t, e'_t)\}
\end{aligned}
$$

Also in this case $\mathbb{M}' = \mathbb{M}'' \uplus \{(e'_s, e'_t)\}$.
Now we check whether G'_{src} is consistent.
If $G'_{\text{src}} = G''_{\text{src}}$ then G_{src} is consistent as G''_{src} is consistent.
Otherwise, we check whether G'_{src} is consistent.

We know G''_{src} and G'_{tgt} preserve (CF). As a result, from the construction $(e'_s, e'_s) \notin G'_{\text{src}}.\text{ecf}$. Hence G'_{src} preserves (CF).

We know G''_{src} preserves (CFJ). Moreover, $G'_{\text{tgt}}.\text{jf}(w_t, e'_t)$ implies $\neg G'_{\text{tgt}}.\text{ecf}(w_t, e'_t)$. As a result, from the construction $\neg G'_{\text{src}}.\text{ecf}(w_s, e'_s)$ where $\mathbb{M}''(w_s, w_t)$ holds. Hence G'_{src} preserves (CFJ).

We know G''_{src} preserves (VISJ). Moreover, $G'_{\text{tgt}}.\text{jf}(w_t, e'_t)$ implies $w_t \in \text{vis}(G'_{\text{tgt}})$. As a result, from the construction $w_s \in \text{vis}(G''_{\text{src}})$ where $\mathbb{M}''(w_s, w_t)$ holds. Hence G'_{src} preserves (VISJ).

We know G''_{src} and G'_{tgt} preserves (ICF). hence following the construction we know if $e'_s \notin G'_{\text{src}}.\mathcal{R}$ then there exists no event e_1 such that $G'_{\text{src}}.\sim(e'_s, e_1)$. Hence G'_{src} preserves (ICF).

We know G''_{src} preserves (ICFJ). Moreover, following the construction of G'_{src} from G''_{src}, $(w_s, w_s) \notin G'_{\text{src}}.\text{jf}; \text{imm}(\text{cf}); G'_{\text{src}}.\text{rf}^{-1}$. Hence G'_{src} preserves (ICFJ).

We know G''_{src} preserves (COH′) and consider there is a $(G'_{\text{src}}.\text{hb}; G'_{\text{src}}.\text{eco}^?)$ cycle. In that case e'_s is part of the $(G'_{\text{src}}.\text{hb}; G'_{\text{src}}.\text{eco}^?)$ cycle. However, following the construction of G'_{src}, in this case, there exists a $(G'_{\text{tgt}}.\text{hb}; G'_{\text{tgt}}.\text{eco}^?)$ cycle. This is not possible as G'_{tgt} is consistent. Hence a contradiction and G'_{src} preserves (COH′). As a result, G'_{src} is consistent.

Thus finally $\mathbb{M}' = \mathbb{M} \uplus \{(e'_s, e'_t)\}$ and $\text{pc}' = \text{pc}$.

(2) Condition to show:the simulation invariant holds between G'_{src} and G'_{tgt}

(a)

$$\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B').\ \exists c_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t)$$

We know this condition holds between G_{src} and G_{tgt}. Hence the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B')$.

(b)

$$\begin{aligned}
&\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A', b_t \in B' \wedge G'_{\text{tgt}}.\text{po}(c_t, b_t) \implies \\
&\exists c_s, a_s, b_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t) \\
&\wedge(\exists a'' \in G'_{\text{src}}.\mathsf{E}.\ a_s.\text{loc} = a''.\text{loc} \wedge a_s.\text{ord} = a''.\text{ord} \\
&\wedge G'_{\text{src}}.\text{po}(c_s, a'') \wedge \text{imm}(G'_{\text{src}}.\text{po})(a'', b_s))
\end{aligned}$$

We know this condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, and $\mathbb{M}'$ the condition holds between G'_{src} and G'_{tgt} where $b_t = e'_t$, $b_s = e'_s$, and $a'' = a'$.

(c)

$$\begin{aligned}
&\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'. \wedge G'_{\text{tgt}}.\text{po}(c_t, a_t) \implies \\
&\exists c_s, a_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge G'_{\text{src}}.\text{po}(c_s, a_s)
\end{aligned}$$

We know this condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$ this condition holds between G'_{src} and G'_{tgt} for all e'_t, e'_s, a'.

(d)

$$\begin{aligned}
&\forall a_t \in A', b_t \in B'.\ \text{imm}(G'_{\text{tgt}}.\text{po})(b_t, a_t) \implies \\
&(\exists c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a', b_s, c_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t) \\
&\wedge \text{imm}(G'_{\text{tgt}}.\text{po})(c_t, b_t) \wedge \text{imm}(G'_{\text{src}}.\text{po})(c_s, a_s) \wedge \text{imm}(G'_{\text{src}}.\text{po})(a_s, b') \\
&\wedge \text{imm}(G'_{\text{src}}.\text{cf})(a_s, a') \wedge \text{imm}(G'_{\text{src}}.\text{po})(a', b_s) \wedge b_s.\text{loc} = b'.\text{loc} \wedge b_s.\text{ord} = b'.\text{ord} \\
&\wedge G'_{\text{src}}.\text{ew}(b_s, b'))
\end{aligned}$$

We know this condition holds between G_{src} and G_{tgt}. The event e'_t is G'_{tgt}.po-maximal and hence $\text{imm}(G'_{\text{tgt}}.\text{po})(b_t, a_t)$ does not hold when $b_t = e'_t$. Hence the condition holds between G'_{src} and G'_{tgt}.

(e)

$$\begin{aligned}&\forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathsf{tgt}}.\mathsf{po}(b_t, c_t) \implies \\ &\exists b_s, b', c_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(b_s, b_t) \land \mathbb{M}'(b', b_t) \\ &\land G'_{\mathsf{src}}.\mathsf{ew}(b_s, b') \land (G'_{\mathsf{src}}.\mathsf{po}(b_s, c_s) \lor G'_{\mathsf{src}}.\mathsf{po}(b', c_s))\end{aligned}$$

We know this condition holds between G_{src} and G_{tgt}. The event e'_t is G'_{tgt}.po-maximal and hence $G'_{\mathsf{tgt}}.\mathsf{po}(b_t, c_t)$ does not hold when $b_t = e'_t$. Hence the condition holds between G'_{src} and G'_{tgt}.

(f)

$$\begin{aligned}&\forall a_t \in A', b_t \in B'.\ G'_{\mathsf{tgt}}.\mathsf{po}(b_t, a_t) \implies \\ &\exists a_s, b_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(b_s, b_t) \land \neg G'_{\mathsf{src}}.\mathsf{po}(b_s, a_s)\end{aligned}$$

We know this condition holds between G_{src} and G_{tgt}. The event e'_t is G'_{tgt}.po-maximal and hence $G'_{\mathsf{tgt}}.\mathsf{po}(b_t, a_t)$ does not hold when $b_t = e'_t$. Hence the condition holds between G'_{src} and G'_{tgt}.

(g)

$$\begin{aligned}&\forall c_t, c'_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\mathsf{tgt}}.\mathsf{po}(c_t, c'_t) \implies \\ &\exists c_s, c'_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(c'_s, c'_t) \land G'_{\mathsf{src}}.\mathsf{po}(c_s, c'_s)\end{aligned}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B')$.

(h)

$$\begin{aligned}&\forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\mathsf{tgt}}.\mathsf{jf}(c_t, a_t) \implies \\ &\exists c_s, a_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(c_s, c_t) \land G'_{\mathsf{src}}.\mathsf{jf}(c_s, a_s)\end{aligned}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B')$ or $e'_t \notin A$.

(i)

$$\begin{aligned}&\forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\mathsf{tgt}}.\mathsf{jf}(a_t, c_t) \implies \\ &\exists c_s, a_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(c_s, c_t) \land G'_{\mathsf{src}}.\mathsf{jf}(a_s, c_s)\end{aligned}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B')$ and $e'_t \notin A$.

(j)

$$\begin{aligned}&\forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathsf{tgt}}.\mathsf{jf}(c_t, b_t) \implies \\ &\exists b_s, c_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(b_s, b_t) \land \mathbb{M}'(c_s, c_t) \land G'_{\mathsf{src}}.\mathsf{jf}(c_s, b_s) \\ &\land (\exists b' \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(b', b_t) \land G'_{\mathsf{src}}.\mathsf{ew}(b_s, b') \implies G'_{\mathsf{src}}.\mathsf{jf}(c_s, b'))\end{aligned}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} where $b_s = e'_t$ and there exists no b' such that $\mathbb{M}'(b_s, b')$.

(k)

$$\begin{aligned}&\forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathsf{tgt}}.\mathsf{jf}(b_t, c_t) \implies \\ &((\exists b_s, c_s \in G'_{\mathsf{src}}.\mathsf{E}.\ (\mathbb{M}'(b_s, b_t) \land \nexists b' \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}(b', b_t) \land G'_{\mathsf{src}}.\mathsf{ew}(b_s, b')) \\ &\qquad\qquad \implies G'_{\mathsf{src}}.\mathsf{jf}(b_s, c_s)) \\ &\lor (\exists b', b_s, c_s \in G'_{\mathsf{src}}.\mathsf{E}.\ (\mathbb{M}'(b_s, b_t) \land \mathbb{M}'(b', b_t) \land \mathbb{M}'(c_s, c_t) \land G'_{\mathsf{src}}.\mathsf{ew}(b_s, b')) \implies \\ &\qquad\qquad G'_{\mathsf{src}}.\mathsf{jf}(b', c_s)))\end{aligned}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} where $b_s = e'_t$ and there exists no $b' \in G'_{\mathrm{src}}.\mathsf{E}$ such that $\mathbb{M}(b', b_t)$ and $G_{\mathrm{src}}.\mathsf{ew}(b_s, b')$ holds.

(l)

$$\forall c_t, c'_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\mathrm{tgt}}.\mathsf{jf}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(c'_s, c'_t) \land G'_{\mathrm{src}}.\mathsf{jf}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$.

(m)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A', b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{mo}(c_t, a_t) \implies$$
$$\exists c_s, a_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(a_s, a_t) \land G'_{\mathrm{src}}.\mathsf{mo}(c_s, a_s)$$

We know the condition holds between G_{src} and G_{tgt}.
Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$ and forall $a_t \in A'.\ \neg\mathbb{M}'(a', a_t)$ holds.

(n)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\mathrm{tgt}}.\mathsf{mo}(a_t, c_t) \implies$$
$$\exists c_s, a_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \land \mathbb{M}'(a_s, a_t) \land G'_{\mathrm{src}}.\mathsf{mo}(a_s, c_s)$$

We know the condition holds between G_{src} and G_{tgt}.
Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$ and $e'_t \notin A'$.

(o)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{mo}(c_t, b_t) \implies$$
$$\exists c_s, b_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \land \mathbb{M}'(b_s, b_t) \land G'_{\mathrm{src}}.\mathsf{mo}(c_s, b_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src} and G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} where $b_t = e'_t$ and $b_s = e'_s$.

(p)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{mo}(b_t, c_t) \implies$$
$$\exists c_s, b_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(b_s, b_t) \land G'_{\mathrm{src}}.\mathsf{mo}(b_s, c_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} where $b_t = e'_t$ and $b_s = e'_s$.

(q)

$$\forall c, c' \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\mathrm{tgt}}.\mathsf{mo}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(c'_s, c'_t) \land G'_{\mathrm{src}}.\mathsf{mo}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}.
Following the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$.

(r)

$$\forall o_s \in G'_{\text{src}}.\mathcal{W}.\ (\nexists o_t \in G'_{\text{tgt}}.\mathsf{E}.\ \mathbb{M}'(o_s, o_t)) \implies \nexists o'_s \in G'_{\text{src}}.\mathsf{E}.\ G'_{\text{src}}.\text{mo}(o_s, o'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds where $o_s = a'$.

(s)

$$\forall c_t, c'_t \in G'_{\text{tgt}}.\mathsf{E}.\ G'_{\text{tgt}}.\text{ew}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(c'_s, c'_t) \wedge G_{\text{src}}.\text{ew}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} where $c_t = e'_t$ or $c'_t = e'_t$ and $c_s = e'_s$ and $c'_s = e'_s$.

Hence the invariant holds between G'_{src} and G'_{tgt}.

(3) Condition to show:
there exists pc' *such that*
$\mathsf{X}'_s.\mathsf{E} = \mathbb{S}'$
$\mathsf{X}'_s.\text{po} = G'_{\text{src}}.\text{po} \cap (\mathbb{S}' \times \mathbb{S}')$
$\mathsf{X}'_s.\text{rf} = G'_{\text{src}}.\text{rf} \cap (\mathbb{S}' \times \mathbb{S}')$
$\mathsf{X}'_s.\text{mo} = G'_{\text{src}}.\text{mo} \cap (\mathbb{S}' \times \mathbb{S}')$
where $\mathbb{S}'(G'_{\text{src}}, \text{pc}') \triangleq \{e \mid e \in G'_{\text{src}}.\mathsf{E} \wedge G'_{\text{src}}.\text{po}^?(e, \text{pc}'(e.\text{tid}))\}$.

We know there exists pc such that
$\mathsf{X}_s.\mathsf{E} = \mathbb{S}$
$\mathsf{X}_s.\text{po} = G_{\text{src}}.\text{po} \cap (\mathbb{S} \times \mathbb{S})$
$\mathsf{X}_s.\text{rf} = G_{\text{src}}.\text{rf} \cap (\mathbb{S} \times \mathbb{S})$
$\mathsf{X}_s.\text{mo} = G_{\text{src}}.\text{mo} \cap (\mathbb{S} \times \mathbb{S})$
where $\mathbb{S}(G_{\text{src}}, \text{pc}) \triangleq \{e \mid e \in G_{\text{src}}.\mathsf{E} \wedge G_{\text{src}}.\text{po}^?(e, \text{pc}(e.\text{tid}))\}$ and $\text{pc}' = \text{pc}$ holds.
In this case $\mathsf{X}'_s = \mathsf{X}_s$.

As a result, $G'_{\text{src}} \sim G'_{\text{tgt}}$ holds.

Case $e'_t \in A$ where $A' = A \uplus \{e'_t\}$:
The construction has two steps: $G_{\text{src}} \longrightarrow G''_{\text{src}} \longrightarrow G'_{\text{src}}$. In G''_{src} we introduce e'_s and in G'_{src} we introduce b'.
In this case $B' = B$, and $G'_{\text{tgt}}.\mathsf{E} = G_{\text{tgt}}.\mathsf{E} \uplus \{e'_t\}$.
Let $c_t \in C$ be the immediate G_{tgt}.po-predecessor of e_t, that is, $\text{imm}(G_{\text{tgt}}.\text{po})(c_t, e_t)$.
In G_{src} the event c_s is the corresponding event of c_t, that is, $\mathbb{M}(c_s, c_t)$.
We also append corresponding event(s) in G_{src} and construct G'_{src}.

(1) Condition to show: G'_{src} *is consistent.*
case. event e_s has an immediate po successor a'' such that $e'_t.\text{lab} = a''.\text{lab}$ and if $e'_t \in \mathcal{R}$ and $G'_{\text{tgt}}.\text{jf}(w_t, e'_t)$ then there exists w_s such that $\mathbb{M}(w_s, w_t)$ and $G_{\text{src}}.\text{jf}(w_s, a'')$.
In this case $e'_s = a''$ and $G''_{\text{src}} = G_{\text{src}}$.
otherwise.

We append an event e_s' in G_{src} by po-extending from e_s and create G''_{src} such that

$$
\begin{aligned}
G''_{\text{src}}.\mathsf{E} =\ & G_{\text{src}}.\mathsf{E} \uplus \{e_s'\} \\
G''_{\text{src}}.\mathsf{po} =\ & (G_{\text{src}}.\mathsf{po} \uplus \{(e_s, e_s') \mid \mathbb{M}(e_s, e_t)\})^+ \\
G''_{\text{src}}.\mathsf{jf} =\ & G_{\text{src}}.\mathsf{jf} \uplus \{(w_s, e_s') \mid (w_s, e_s') \in (G''_{\text{src}}.\mathcal{W} \times G''_{\text{src}}.\mathcal{R}) \\
& \wedge G'_{\text{tgt}}.\mathsf{jf}(w_t, e_t') \wedge \mathbb{M}(w_s, w_t)\} \\
G''_{\text{src}}.\mathsf{mo} =\ & G_{\text{src}}.\mathsf{mo} \uplus \{(w_s, e_s') \mid (w_s, e_s') \in (G''_{\text{src}}.\mathcal{W} \times G''_{\text{src}}.\mathcal{W}) \\
& \wedge \mathbb{M}(w_s, w_t) \wedge G'_{\text{tgt}}.\mathsf{mo}(w_t, e_t')\} \\
& \uplus \{(e_s', w_s) \mid (e_s', w_s) \in (G''_{\text{src}}.\mathcal{W} \times G''_{\text{src}}.\mathcal{W}) \\
& \wedge \mathbb{M}(w_s, w_t) \wedge G'_{\text{tgt}}.\mathsf{mo}(w_t, e_t')\} \\
G''_{\text{src}}.\mathsf{ew} =\ & G_{\text{src}}.\mathsf{ew} \uplus \{(w_s, e_s'), (e_s', w_s) \mid (w_s, e_s') \in (G''_{\text{src}}.\mathcal{W}_{\sqsubseteq\text{RLX}} \times G''_{\text{src}}.\mathcal{W}_{\sqsubseteq\text{RLX}}) \\
& \wedge \mathbb{M}(w_s, w_t) \wedge G'_{\text{tgt}}.\mathsf{ew}(w_t, e_t')\}
\end{aligned}
$$

Also in this case $\mathbb{M}'' = \mathbb{M} \uplus \{(e_s', e_t')\}$.
Now we check whether G''_{src} is consistent.
We know that $G_{\text{tgt}} \sim G_{\text{src}}$ and hence G_{src} and G_{tgt} are consistent. Now we check whether G''_{src} is consistent.
If $G''_{\text{src}} = G_{\text{src}}$ then G''_{src} is consistent as G_{src} is consistent.
Otherwise.

We know that G_{src} preserves (ICFJ). Also from the construction of G''_{src}, we know there is no $G''_{\text{src}}.\mathsf{jf}(e_s', -)$. Hence G''_{src} preserves (ICFJ).

We know that G_{src} preserves (CF), (CFJ), (VISJ), (CFJ). Also $G'_{\text{tgt}}.\mathsf{jf}(w_t, e_t')$ implies $e_s' \in \mathcal{R}$, $w_t \in \text{vis}(G'_{\text{tgt}})$ and $\neg G'_{\text{tgt}}.\text{ecf}(w_t, e_t')$, and $\mathbb{M}(w_s, w_t)$ holds. Following the construction, $w_s \in \text{vis}(G''_{\text{src}})$, $\neg G''_{\text{src}}.\text{ecf}(w_s, e_s')$ holds. Hence G''_{src} preserves (CF), (CFJ), (VISJ), (ICF).

We know G_{src} preserves (COH′). Consider there is $(G''_{\text{src}}.\mathsf{hb}; G''_{\text{src}}.\mathsf{eco}^?)$ cycle in G''_{src} and e_s' is a part of this cycle. In that case there is a $(G'_{\text{tgt}}.\mathsf{hb}; G'_{\text{tgt}}.\mathsf{eco}^?)$ cycle in G'_{tgt} and e_t' is a part of the cycle. However, G'_{tgt} preserves (COH′) and hence there is no $(G'_{\text{tgt}}.\mathsf{hb}; G'_{\text{tgt}}.\mathsf{eco}^?)$ cycle. Hence a contradiction and G''_{src} preserves (COH′).
As a result, G''_{src} is consistent.
Next, we construct G'_{src} from G''_{src} where we identify or create e_s'.
case. There exists e_s' where $e_s'.\text{lab} = e_t'.\text{lab}$ and if $e_s', e_t' \in \mathcal{R}$, then $G''_{\text{src}}.\mathsf{jf}(w_s, e_s')$ and $G''_{\text{src}}.\mathsf{jf}(w_t, e_s')$ and $\mathbb{M}''(w_s, w_t)$ hold.
In this case $G'_{\text{src}} = G''_{\text{src}}$.
Otherwise. We append such a $e_s' = b'$ and thus

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} =& G''_{\mathsf{src}}.\mathsf{E} \uplus \{b' \mid b'.\mathsf{lab} = e_t.\mathsf{lab}\}\\
G'_{\mathsf{src}}.\mathsf{po} =& (G''_{\mathsf{src}}.\mathsf{po} \uplus \{(e'_s, b')\})^+\\
G'_{\mathsf{src}}.\mathsf{jf} =& G''_{\mathsf{src}}.\mathsf{jf} \uplus \{(w_s, b') \mid (w_s, b') \in (G'_{\mathsf{src}}.\mathcal{W} \times G'_{\mathsf{src}}.\mathcal{R}) \land G'_{\mathsf{tgt}}.\mathsf{jf}(w_t, e_t)\\
& \land \mathbb{M}''(w_s, w_t) \land \neg G''_{\mathsf{src}}.\mathsf{cf}(w_s, e_s)\}\\
G'_{\mathsf{src}}.\mathsf{mo} =& G''_{\mathsf{src}}.\mathsf{mo} \uplus \{(w_s, b') \mid (w_s, b') \in (G'_{\mathsf{src}}.\mathcal{W} \times G'_{\mathsf{src}}.\mathcal{W})\\
& \land \mathbb{M}''(w_s, w_t) \land G'_{\mathsf{tgt}}.\mathsf{mo}(w_t, e_t) \land \neg G''_{\mathsf{src}}.\mathsf{cf}(w_s, b')\}\\
& \uplus \{(b', w_s) \mid (b', w_s) \in (G'_{\mathsf{src}}.\mathcal{W} \times G'_{\mathsf{src}}.\mathcal{W})\\
& \land \mathbb{M}''(w_s, w_t) \land G'_{\mathsf{tgt}}.\mathsf{mo}(e_t, w_t) \land \neg G''_{\mathsf{src}}.\mathsf{cf}(w_s, b')\}\\
G'_{\mathsf{src}}.\mathsf{ew} =& G''_{\mathsf{src}}.\mathsf{ew}\\
& \uplus \{(w_s, b'), (b', w_s) \mid (w_s, b') \in (G'_{\mathsf{src}}.\mathcal{W}_{\sqsubseteq\mathrm{RLX}} \times G'_{\mathsf{src}}.\mathcal{W}_{\sqsubseteq\mathrm{RLX}}) \land \mathbb{M}''(w_s, e_t)\}
\end{aligned}
$$

Also in this case $\mathbb{M}' = \mathbb{M}'' \uplus \{(e'_s, e'_t)\}$.
Now we check whether G'_{src} is consistent.
If $G'_{\mathsf{src}} = G''_{\mathsf{src}}$ then G'_{src} is consistent as G''_{src} is consistent.
Otherwise we check the consistency of G'_{src}.

We know G''_{src} and G'_{tgt} preserve (CF). As a result, from the construction $(e'_s, e'_s) \notin G'_{\mathsf{src}}.\mathsf{ecf}$. Hence G'_{src} preserves (CF).

We know G''_{src} preserves (CFJ). Moreover, $G'_{\mathsf{tgt}}.\mathsf{jf}(w_t, e'_t)$ implies $\neg G'_{\mathsf{tgt}}.\mathsf{ecf}(w_t, e'_t)$. As a result, from the construction $\neg G'_{\mathsf{src}}.\mathsf{ecf}(w_s, e'_s)$ where $\mathbb{M}''(w_s, w_t)$ holds. Hence G'_{src} preserves (CFJ).

We know G''_{src} preserves (CFJ). Moreover, $G'_{\mathsf{tgt}}.\mathsf{jf}(w_t, e_t)$ implies $\neg G'_{\mathsf{tgt}}.\mathsf{cf}(w_t, e_t)$. As a result, from the construction $\neg G'_{\mathsf{src}}.\mathsf{cf}(w_s, b')$ where $\mathbb{M}''(w_s, w_t)$ holds. Hence G'_{src} preserves (CFJ).

We know G''_{src} preserves (VISJ). Moreover, $G'_{\mathsf{tgt}}.\mathsf{jf}(w_t, e_t)$ implies $w_t \in \mathsf{vis}(G'_{\mathsf{tgt}})$. As a result, from the construction $w_s \in \mathsf{vis}(G'_{\mathsf{src}})$ where $\mathbb{M}''(w_s, w_t)$ holds. Hence G'_{src} preserves (VISJ).

We know G''_{src} and G'_{tgt} preserves (ICF). Hence following the construction we know that G'_{src} preserves (ICF).

We know that G''_{src} preserves (ICFJ). Also from the construction of G'_{src}, we know there is no $G'_{\mathsf{src}}.\mathsf{jf}(e'_s, -)$. Hence G'_{src} preserves (ICFJ).

We know G''_{src} preserves (COH′) and consider there is a $(G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?)$ cycle. In that case b' is part of the $(G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?)$ cycle. However, following the construction of G'_{src}, in this case, there exists a $(G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^?)$ cycle. This is not possible as G'_{tgt} is consistent. Hence a contradiction and G'_{src} preserves (COH′). As a result, G'_{src} is consistent.

Thus finally $\mathbb{M}' = \mathbb{M} \uplus \{(e'_s, e'_t), (b', e_t)\}$ and $\mathsf{pc}' = \mathsf{pc}[e_s.\mathsf{tid} \mapsto b']$.

(2) Condition to show: *the simulation invariant holds between G'_{src} and G'_{tgt}*

(a)

$$\forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B').\ \exists c_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t)$$

In this case $e'_t, e_t \notin G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B')$. Hence the condition holds.

(b)

$$
\begin{aligned}
&\forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A', b_t \in B' \land G'_{\mathsf{tgt}}.\mathsf{po}(c_t, b_t) \implies\\
&\exists c_s, a_s, b_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(b_s, b_t)\\
&\land(\exists a'' \in G'_{\mathsf{src}}.\mathsf{E}.\ a_s.\mathsf{loc} = a''.\mathsf{loc} \land a_s.\mathsf{ord} = a''.\mathsf{ord}\\
&\land G'_{\mathsf{src}}.\mathsf{po}(c_s, a'') \land \mathsf{imm}(G'_{\mathsf{src}}.\mathsf{po})(a'', b_s))
\end{aligned}
$$

We know this condition holds in G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, and $\mathbb{M}'$ the condition holds between G'_{src} and G'_{tgt} where $e_t \notin G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B')$ and $e'_t \notin B'$.

(c)

$$\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'. \wedge G'_{\text{tgt}}.\mathsf{po}(c_t, a_t) \implies$$
$$\exists c_s, a_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge G'_{\text{src}}.\mathsf{po}(c_s, a_s)$$

We know this condition holds in G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$ this condition holds between G'_{src} and G'_{tgt} for $a_t = e'_t$ and $a_s = e'_s$.

(d)

$$\forall a_t \in A', b_t \in B'.\ \mathsf{imm}(G'_{\text{tgt}}.\mathsf{po})(b_t, a_t) \implies$$
$$(\exists c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a', b_s, c_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t)$$
$$\wedge \mathsf{imm}(G'_{\text{tgt}}.\mathsf{po})(c_t, b_t) \wedge \mathsf{imm}(G'_{\text{src}}.\mathsf{po}, c_s, a_s) \wedge \mathsf{imm}(G'_{\text{src}}.\mathsf{po}, a_s, b')$$
$$\wedge G'_{\text{src}}.\mathsf{cf}(a_s, a') \wedge \mathsf{imm}(G'_{\text{src}}.\mathsf{po})(a', b_s)$$
$$\wedge b_s.\mathsf{loc} = b'.\mathsf{loc} \wedge b_s.\mathsf{ord} = b'.\mathsf{ord} \wedge G'_{\text{src}}.\mathsf{ew}(b_s, b'))$$

We know this condition holds in G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$ we have $b_t = e_t$, $a_t = e'_t$, $a_s = e'_s$, $b_s = e_s$ and from the construction we know there exists such an $a' \in G_{\text{src}}.\mathsf{E}$ so that $\mathsf{imm}(G_{\text{src}}.\mathsf{po})(a', b_s)$ holds. In this case $\mathbb{M}'(e_s, e_t)$, $\mathbb{M}'(b', e_t)$, and $G'_{\text{tgt}}.\mathsf{ew}(e_s, b')$ hold.
As a result, this condition holds between G'_{src} and G'_{tgt}.

(e)

$$\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\text{tgt}}.\mathsf{po}(b_t, c_t) \implies$$
$$\exists b_s, b'', c_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(b_s, b_t) \wedge \mathbb{M}'(b'', b_t)$$
$$\wedge G'_{\text{src}}.\mathsf{ew}(b_s, b'') \wedge (G'_{\text{src}}.\mathsf{po}(b_s, c_s) \vee G'_{\text{src}}.\mathsf{po}(b'', c_s))$$

We know this condition holds in G_{src} and G_{tgt}.
Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$ we know $b', e_t \notin G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B')$. Hence the condition holds between G'_{src} and G'_{tgt}.

(f)

$$\forall a_t \in A', b_t \in B'.\ G'_{\text{tgt}}.\mathsf{po}(b_t, a_t) \implies$$
$$\exists a_s, b_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t) \wedge \neg G'_{\text{src}}.\mathsf{po}(b_s, a_s)$$

We know the condition holds between G'_{src} and G'_{tgt}.
Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$ for $b_t = e_t$, $a_t = e'_t$, $a_s = e'_s$, $b_s = b'$ the condition holds between G'_{src} and G'_{tgt}.

(g)

$$\forall c_t, c'_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\text{tgt}}.\mathsf{po}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(c'_s, c'_t) \wedge G'_{\text{src}}.\mathsf{po}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}. In this case $e'_t \notin G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B')$. Hence the condition holds between G'_{src} and G'_{tgt}.

(h)

$$\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\text{tgt}}.\mathsf{jf}(c_t, a_t) \implies$$
$$\exists c_s, a_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(c_s, c_t) \wedge G'_{\text{src}}.\mathsf{jf}(c_s, a_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds for $a_t = e'_t$, $a_s = e'_s$ between G'_{src} and G'_{tgt}.

(i)

$$\begin{array}{l}\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\mathrm{tgt}}.\mathsf{jf}(a_t, c_t) \implies \\ \exists c_s, a_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(c_s, c_t) \wedge G'_{\mathrm{src}}.\mathsf{jf}(a_s, c_s)\end{array}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, for $a_t = e'_t$ there is no outgoing edge from e'_t. Hence the condition holds between G'_{src} and G'_{tgt}.

(j)

$$\begin{array}{l}\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{jf}(c_t, b_t) \implies \\ \exists b_s, c_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(b_s, b_t) \wedge \mathbb{M}'(c_s, c_t) \wedge G'_{\mathrm{src}}.\mathsf{jf}(c_s, b_s) \\ \wedge(\exists b' \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(b', b_t) \wedge G'_{\mathrm{src}}.\mathsf{ew}(b_s, b') \implies G'_{\mathrm{src}}.\mathsf{jf}(c_s, b'))\end{array}$$

We know the condition holds between G_{src} and G_{tgt}. In this case the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin B'$.

(k)

$$\begin{array}{l}\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{jf}(b_t, c_t) \implies \\ ((\exists b_s, c_s \in G'_{\mathrm{src}}.\mathsf{E}.\ (\mathbb{M}'(b_s, b_t) \wedge \nexists b' \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(b', b_t) \wedge G'_{\mathrm{src}}.\mathsf{ew}(b_s, b')) \implies \\ \qquad G'_{\mathrm{src}}.\mathsf{jf}(b_s, c_s)) \\ \wedge(\exists b', b_s, c_s \in G'_{\mathrm{src}}.\mathsf{E}.\ (\mathbb{M}'(b_s, b_t) \wedge \mathbb{M}'(b', b_t) \wedge \mathbb{M}'(c_s, c_t) \wedge G'_{\mathrm{src}}.\mathsf{ew}(b_s, b')) \implies \\ \qquad G'_{\mathrm{src}}.\mathsf{jf}(b', c_s)))\end{array}$$

We know the condition holds between G_{src} and G_{tgt}. In this case the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin B'$ and $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$.

(l)

$$\begin{array}{l}\forall c_t, c'_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\mathrm{tgt}}.\mathsf{jf}(c_t, c'_t) \implies \\ \exists c_s, c'_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(c'_s, c'_t) \wedge G'_{\mathrm{src}}.\mathsf{jf}(c_s, c'_s)\end{array}$$

We know the condition holds between G_{src} and G_{tgt}. In this case $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$. Hence the condition holds between G'_{src} and G'_{tgt}.

(m)

$$\begin{array}{l}\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A', b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{mo}(c_t, a_t) \implies \\ \exists c_s, a_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge G'_{\mathrm{src}}.\mathsf{mo}(c_s, a_s)\end{array}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, for $a_t = e'_t$ and $a_s = e'_s$ the condition holds between G'_{src} and G'_{tgt}.

(n)

$$\begin{array}{l}\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\mathrm{tgt}}.\mathsf{mo}(a_t, c_t) \implies \\ \exists c_s, a_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge G'_{\mathrm{src}}.\mathsf{mo}(a_s, c_s)\end{array}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, for $a_t = e'_t$ and $a_s = e'_s$ the condition holds between G'_{src} and G'_{tgt}.

(o)

$$\begin{array}{l}\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{mo}(c_t, b_t) \implies \\ \exists c_s, b_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \wedge \mathbb{M}'(b_s, b_t) \wedge G'_{\mathrm{src}}.\mathsf{mo}(c_s, b_s)\end{array}$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src} and G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin B'$ and $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$.

(p)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathrm{tgt}}.\mathrm{mo}(b_t, c_t) \implies$$
$$\exists c_s, b_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(b_s, b_t) \wedge G'_{\mathrm{src}}.\mathrm{mo}(b_s, c_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src} and G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $e'_t \notin B'$ and $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$.

(q)

$$\forall c, c' \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\mathrm{tgt}}.\mathrm{mo}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(c'_s, c'_t) \wedge G'_{\mathrm{src}}.\mathrm{mo}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}. In this case $e'_t \notin G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B')$. Hence the condition holds between G'_{src} and G'_{tgt}.

(r)

$$\forall o_s \in G'_{\mathrm{src}}.\mathcal{W}.\ (\nexists o_t \in G'_{\mathrm{tgt}}.\mathsf{E}.\ \mathbb{M}'(o_s, o_t)) \implies$$
$$\nexists o'_s \in G'_{\mathrm{src}}.\mathsf{E}.\ G'_{\mathrm{src}}.\mathrm{mo}(o_s, o'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src} and G'_{tgt}, $\mathbb{M}'$, $(e'_s, e_t), (b', e_s) \in \mathbb{M}'$. Hence the condition holds between G'_{src} and G'_{tgt}.

(s)

$$\forall c_t, c'_t \in G'_{\mathrm{tgt}}.\mathsf{E}.\ G'_{\mathrm{tgt}}.\mathrm{ew}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(c'_s, c'_t) \wedge G_{\mathrm{src}}.\mathrm{ew}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src} and G'_{tgt}, $\mathbb{M}'$ the condition holds between G'_{src} and G'_{tgt} as $G'_{\mathrm{tgt}}.\mathrm{ew} = G_{\mathrm{tgt}}.\mathrm{ew}$.

Hence the invariant holds between G'_{src} and G'_{tgt}.

(3) Condition to show:
there exists pc′ *such that*
$\mathsf{X}'_s.\mathsf{E} = \mathbb{S}'$
$\mathsf{X}'_s.\mathrm{po} = G'_{\mathrm{src}}.\mathrm{po} \cap (\mathbb{S}' \times \mathbb{S}')$
$\mathsf{X}'_s.\mathrm{rf} = G'_{\mathrm{src}}.\mathrm{rf} \cap (\mathbb{S}' \times \mathbb{S}')$
$\mathsf{X}'_s.\mathrm{mo} = G'_{\mathrm{src}}.\mathrm{mo} \cap (\mathbb{S}' \times \mathbb{S}')$
where $\mathbb{S}'(G'_{\mathrm{src}}, \mathrm{pc}') \triangleq \{e \mid e \in G'_{\mathrm{src}}.\mathsf{E} \wedge G'_{\mathrm{src}}.\mathrm{po}^?(e, \mathrm{pc}'(e.\mathrm{tid}))\}$.
If $e'_t \notin \mathsf{X}'_t$ then $\mathsf{X}'_t = \mathsf{X}_t$. In this case $\mathrm{pc}' = \mathrm{pc}$, $\mathbb{S}' = \mathbb{S}$, and $\mathsf{X}'_s = \mathsf{X}_s$.

Otherwise, when $e'_t \in X'_t$ then X'_t is an extension of X_t, that is,

$$\begin{aligned}
X'_t.E =& X_t.E \uplus \{e_t, e'_t\} \\
X'_t.po =& (X_t.po \uplus \{(a, e_t) \mid a \in X_t.E \wedge G'_{tgt}.po(a, e_t)\} \\
&\uplus \{(a, e'_t) \mid a \in X_t.E \wedge G'_{tgt}.po(a, e'_t)\} \uplus \{(e_t, e'_t)\})^+ \\
X'_t.rf =& X_t.rf \uplus \{(a, e_t) \mid a \in X_t.E \wedge G'_{tgt}.rf(a, e_t)\} \\
&\uplus \{(a, e'_t) \mid a \in X_t.E \wedge G'_{tgt}.rf(a, e'_t)\} \\
&\uplus \{(e_t, a) \mid a \in X_t.E \wedge G'_{tgt}.rf(e_t, a)\} \\
&\uplus \{(e'_t, a) \mid a \in X_t.E \wedge G'_{tgt}.rf(e'_t, a)\} \\
X'_t.mo =& X_t.mo \uplus \{(a, e_t) \mid a \in X_t.E \wedge G'_{tgt}.mo(a, e_t)\} \\
&\uplus \{(a, e'_t) \mid a \in X_t.E \wedge G'_{tgt}.mo(a, e'_t)\} \\
&\uplus \{(e_t, a) \mid a \in X_t.E \wedge G'_{tgt}.mo(e_t, a)\} \\
&\uplus \{(e'_t, a) \mid a \in X_t.E \wedge G'_{tgt}.mo(e'_t, a)\}
\end{aligned}$$

We also know that the X_t and X_s are related as follows.
$X_s.E = X_t.E$
$X_s.po = \{(a_s, b_s) \mid \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge X_t.po(a_t, b_t)\}$
$X_s.rf = \{(a_s, b_s) \mid \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge X_t.rf(a_t, b_t)\}$
$X_s.mo = \{(a_s, b_s) \mid \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge X_t.mo(a_t, b_t)\}$

Source Execution Extraction.
From X'_t we derive X'_s and relate X'_s to X_s
$X'_s.E = X'_t.E = X_t.E \uplus \{e_t, e'_t\} = X_s.E \uplus \{e_t, e'_t\}$

$X'_s.po = \{(a_s, b_s) \mid X'_t.po(a_t, b_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t)\}$
$\implies X'_s.po = \{(a_s, b_s) \mid X_t.po(a_t, b_t) \wedge \mathbb{M}'(a_s, b_s) \wedge \mathbb{M}'(b_s, b_t)\}$
$\cup \{(a_s, e'_s) \mid X'_t.po(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(a_s, b') \mid X'_t.po(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e_s, e_t)\}$
$\cup \{(e'_s, b') \mid X'_t.po(e_t, e'_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(b', e_t)\}$
$\implies X'_s.po = X_s.po$
$\cup \{(a_s, e'_s) \mid X'_t.po(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(a_s, b') \mid X'_t.po(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e_s, e_t)\}$
$\cup \{(e'_s, b') \mid X'_t.po(e_t, e'_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(b', e_t)\}$

$X'_s.rf = \{(a_s, b_s) \mid X'_t.rf(a_t, b_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t)\}$
$\implies X'_s.rf = \{(a_s, b_s) \mid X_t.rf(a_t, b_t) \wedge \mathbb{M}'(a_s, b_s) \wedge \mathbb{M}'(b_s, b_t)\}$
$\cup \{(a_s, e'_s) \mid X'_t.rf(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(a_s, b') \mid X'_t.rf(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b', e_t)\}$
$\cup \{(e'_s, a_s) \mid X'_t.rf(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\cup \{(b', a_s) \mid X'_t.rf(e_t, a_t) \wedge \mathbb{M}'(b', e_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\implies X'_s.rf = X_s.rf$
$\cup \{(a_s, e'_s) \mid X'_t.rf(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(a_s, b') \mid X'_t.rf(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b', e_t)\}$
$\cup \{(e'_s, a_s) \mid X'_t.rf(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\cup \{(b', a_s) \mid X'_t.rf(e_t, a_t) \wedge \mathbb{M}'(b', e_t) \wedge \mathbb{M}'(a_s, a_t)\}$

$X'_s.mo = \{(a_s, b_s) \mid X'_t.mo(a_t, b_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t)\}$
$\implies X'_s.mo = \{(a_s, b_s) \mid X_t.mo(a_t, b_t) \wedge \mathbb{M}'(a_s, b_s) \wedge \mathbb{M}'(b_s, b_t)\}$
$\cup \{(a_s, e'_s) \mid X'_t.mo(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$

$\cup\, \{(a_s, b') \mid \mathsf{X}'_t.\mathsf{mo}(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b', e_t)\}$
$\cup\, \{(e'_s, a_s) \mid \mathsf{X}'_t.\mathsf{mo}(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\cup\, \{(b', a_s) \mid \mathsf{X}'_t.\mathsf{mo}(e_t, a_t) \wedge \mathbb{M}'(b', e_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\implies \mathsf{X}'_s.\mathsf{mo} = \mathsf{X}_s.\mathsf{mo}$
$\cup\, \{(a_s, e'_s) \mid \mathsf{X}'_t.\mathsf{mo}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup\, \{(a_s, b') \mid \mathsf{X}'_t.\mathsf{mo}(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b', e_t)\}$
$\cup\, \{(e'_s, a_s) \mid \mathsf{X}'_t.\mathsf{mo}(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\cup\, \{(b', a_s) \mid \mathsf{X}'_t.\mathsf{mo}(e_t, a_t) \wedge \mathbb{M}'(b', e_t) \wedge \mathbb{M}'(a_s, a_t)\}$

In this case $\mathrm{pc}' = \mathrm{pc}[b'.\mathrm{tid} \mapsto b']$ and hence
$\mathbb{S}' = \mathbb{S} \uplus \{e'_s, b'\}$.
Now we relate X'_s and $\mathbb{S}'$.
$\mathsf{X}'_s.\mathsf{E} = \mathsf{X}_s.\mathsf{E} \uplus \{e'_s, b'\} = \mathbb{S} \uplus \{e'_s, b'\} = \mathbb{S}'$

We already have
$\mathsf{X}'_s.\mathrm{po} = \mathsf{X}_s.\mathrm{po}$
$\cup\, \{(a_s, e'_s) \mid \mathsf{X}'_t.\mathrm{po}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup\, \{(a_s, b') \mid \mathsf{X}'_t.\mathrm{po}(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e_s, e_t)\}$
$\cup\, \{(e'_s, b') \mid \mathsf{X}'_t.\mathrm{po}(e_t, e'_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(b', e_t)\}$
$\implies \mathsf{X}'_s.\mathrm{po} = G_{\mathrm{src}}.\mathrm{po} \cap (\mathbb{S} \times \mathbb{S}) \cup \{G'_{\mathrm{src}}.\mathrm{po}(a_s, e'_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\cup\, \{(a_s, b') \mid a_s, b' \in \mathbb{S}'\} \cup \{(e'_s, b') \mid e'_s, b' \in \mathbb{S}'\}$
$\implies \mathsf{X}'_s.\mathrm{po} = G'_{\mathrm{src}}.\mathrm{po} \cap (\mathbb{S}' \times \mathbb{S}')$

We already have
$\mathsf{X}'_s.\mathsf{rf} = \mathsf{X}_s.\mathsf{rf}$
$\cup\, \{(a_s, e'_s) \mid \mathsf{X}'_t.\mathsf{rf}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup\, \{(a_s, b') \mid \mathsf{X}'_t.\mathsf{rf}(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b', e_t)\}$
$\cup\, \{(e'_s, a_s) \mid \mathsf{X}'_t.\mathsf{rf}(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\cup\, \{(b', a_s) \mid \mathsf{X}'_t.\mathsf{rf}(e_t, a_t) \wedge \mathbb{M}'(b', e_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\implies \mathsf{X}'_s.\mathsf{rf} = G_{\mathrm{src}}.\mathsf{rf} \cap (\mathbb{S} \times \mathbb{S}) \cup \{G'_{\mathrm{src}}.\mathsf{rf}(a_s, e'_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\cup\, \{G'_{\mathrm{src}}.\mathsf{rf}(a_s, b') \mid a_s, b' \in \mathbb{S}'\}$
$\cup\, \{G'_{\mathrm{src}}.\mathsf{rf}(e'_s, a_s) \mid a_s, e_s \in \mathbb{S}'\} \cup \{G'_{\mathrm{src}}.\mathsf{rf}(b', a_s) \mid a_s, b' \in \mathbb{S}'\}$
$\implies \mathsf{X}'_s.\mathsf{rf} = G'_{\mathrm{src}}.\mathsf{rf} \cap (\mathbb{S}' \times \mathbb{S}')$

We already have
$\mathsf{X}'_s.\mathsf{mo} = \mathsf{X}_s.\mathsf{mo}$
$\cup\, \{(a_s, e'_s) \mid \mathsf{X}'_t.\mathsf{mo}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup\, \{(a_s, b') \mid \mathsf{X}'_t.\mathsf{mo}(a_t, e_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b', e_t)\}$
$\cup\, \{(e'_s, a_s) \mid \mathsf{X}'_t.\mathsf{mo}(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\cup\, \{(b', a_s) \mid \mathsf{X}'_t.\mathsf{mo}(e_t, a_t) \wedge \mathbb{M}'(b', e_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\implies \mathsf{X}'_s.\mathsf{mo} = G_{\mathrm{src}}.\mathsf{mo} \cap (\mathbb{S} \times \mathbb{S})$
$\cup\, \{G'_{\mathrm{src}}.\mathsf{mo}(a_s, e'_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\cup\, \{G'_{\mathrm{src}}.\mathsf{mo}(a_s, b') \mid a_s, b' \in \mathbb{S}'\}$
$\cup\, \{G'_{\mathrm{src}}.\mathsf{mo}(e'_s, a_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\cup\, \{G'_{\mathrm{src}}.\mathsf{mo}(b', a_s) \mid a_s, b' \in \mathbb{S}'\}$
$\implies \mathsf{X}'_s.\mathsf{mo} = G'_{\mathrm{src}}.\mathsf{mo} \cap (\mathbb{S}' \times \mathbb{S}')$

As a result, $G'_{\mathrm{src}} \sim G'_{\mathrm{tgt}}$.

Case $e'_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A', B')$ where $A' = A$ and $B' = B$:
In this case $G'_{\mathrm{tgt}}.\mathsf{E} = G_{\mathrm{tgt}}.\mathsf{E} \uplus \{e'_t\}$.
In G_{src} e_s is the corresponding event of e_t, that is, $\mathbb{M}(e_s, e_t)$.

We also append corresponding event in G_{src} and construct G'_{src}.

(1) Condition to show: *G'_{src} is consistent.*
Two possibilities: (1) either e_s is po-maximal or (2) there exists an event e''_s such that $\mathsf{imm}(G_{\mathsf{src}}.\mathsf{po})(e_s, e''_s)$ and e''_s is $G_{\mathsf{src}}.\mathsf{po}$ maximal.
Let the maximal event be e_m.
We append an event e'_s in G_{src} by po-extending from e_m and create G'_{src} such that

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} =& G_{\mathsf{src}}.\mathsf{E} \uplus \{e'_s\} \\
G'_{\mathsf{src}}.\mathsf{po} =& (G_{\mathsf{src}}.\mathsf{po} \uplus \{(e_m, e'_s)\})^+ \\
G'_{\mathsf{src}}.\mathsf{jf} =& G_{\mathsf{src}}.\mathsf{jf} \uplus \{(w_s, e'_s) \mid (w_s, e'_s) \in (G'_{\mathsf{src}}.\mathcal{W} \times G'_{\mathsf{src}}.\mathcal{R}) \\
& \land \mathbb{M}(w_s, w_t) \land G'_{\mathsf{tgt}}.\mathsf{jf}(w_t, e'_t) \land \neg G'_{\mathsf{src}}.\mathsf{cf}(w_s, e'_s)\} \\
G'_{\mathsf{src}}.\mathsf{mo} =& G_{\mathsf{src}}.\mathsf{mo} \uplus \{(w_s, e'_s) \mid (w_s, e'_s) \in (G'_{\mathsf{src}}.\mathcal{W} \times G'_{\mathsf{src}}.\mathcal{W}) \\
& \land \mathbb{M}(w_s, w_t) \land G'_{\mathsf{tgt}}.\mathsf{mo}(w_t, e_t) \land \neg G'_{\mathsf{src}}.\mathsf{cf}(w_s, e'_s)\} \\
& \uplus \{(e'_s, w_s) \mid (e'_s, w_s) \in (G'_{\mathsf{src}}.\mathcal{W} \times G'_{\mathsf{src}}.\mathcal{W}) \\
& \land \mathbb{M}(w_s, w_t) \land G'_{\mathsf{tgt}}.\mathsf{mo}(e_t, w_t) \land \neg G'_{\mathsf{src}}.\mathsf{cf}(w_s, e'_s)\} \\
G'_{\mathsf{src}}.\mathsf{ew} =& G_{\mathsf{src}}.\mathsf{ew} \uplus \{(w_s, e'_s), (e'_s, w_s) \mid (w_s, e'_s) \in (G'_{\mathsf{src}}.\mathcal{W}_{\sqsubseteq \mathrm{RLX}} \times G'_{\mathsf{src}}.\mathcal{W}_{\sqsubseteq \mathrm{RLX}}) \\
& \land \mathbb{M}(w_s, w_t) \land G'_{\mathsf{tgt}}.\mathsf{ew}(w_t, e_t)\}
\end{aligned}
$$

Also in this case $\mathbb{M}' = \mathbb{M} \uplus \{(e'_s, e'_t)\}$.
Now we check whether G'_{src} is consistent.

We know G_{src}, G'_{tgt} are consistent hence satisfy (ICFJ). Hence from definition of G'_{src} and $\mathbb{M}'$ we know that G'_{src} satisfies (ICFJ).

We know G_{src}, G'_{tgt} are consistent hence satisfy (ICF). Hence following the definition of G'_{src}, and $\mathbb{M}'$ we know G'_{src} preserves (ICF).

We know that G_{src} preserves (CF), (CFJ), (VISJ). Also $G'_{\mathsf{tgt}}.\mathsf{jf}(w_t, e'_t)$ implies $w_t \in \mathsf{vis}(G'_{\mathsf{tgt}})$ and $\neg G'_{\mathsf{tgt}}.\mathsf{ecf}(w_t, e'_t)$, and $\mathbb{M}(w_s, w_t)$ holds. Following the construction, $w_s \in \mathsf{vis}(G'_{\mathsf{src}})$ as well as $\neg G'_{\mathsf{src}}.\mathsf{ecf}(w_s, e'_s)$ hold. Hence G'_{src} preserves (CF), (CFJ), (VISJ).

We know G_{src} preserves (COH′). Consider there is $(G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?)$ cycle in G'_{src} and e'_s is a part of this cycle. In that case there is a $(G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^?)$ cycle in G'_{tgt} and e'_t is a part of the cycle. However, G'_{tgt} preserves (COH′) and hence there is no $(G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^?)$ cycle. Hence a contradiction and G'_{src} preserves (COH′).
As a result, G'_{src} is consistent.
Thus finally $\mathbb{M}' = \mathbb{M} \uplus \{(e'_s, e'_t)\}$ and $\mathsf{pc}' = \mathsf{pc}[e_s.\mathsf{tid} \mapsto e'_s]$.

(2) Condition to show: *the simulation invariant holds between G'_{src} and G'_{tgt}*

(a)

$$\forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B').\ \exists c_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t)$$

We know this condition holds in G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, and $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} as $\mathbb{M}'(e'_s, e'_t)$ holds.

(b)

$$
\begin{aligned}
& \forall c_t \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A', b_t \in B' \land G'_{\mathsf{tgt}}.\mathsf{po}(c_t, b_t) \implies \\
& \exists c_s, a_s, b_s \in G'_{\mathsf{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(b_s, b_t) \\
& \land(\exists a'' \in G'_{\mathsf{src}}.\mathsf{E}.\ a_s.\mathsf{loc} = a''.\mathsf{loc} \land a_s.\mathsf{ord} = a''.\mathsf{ord} \\
& \land G'_{\mathsf{src}}.\mathsf{po}(c_s, a'') \land \mathsf{imm}(G'_{\mathsf{src}}.\mathsf{po})(a'', b_s))
\end{aligned}
$$

We know this condition holds in G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, and $\mathbb{M}'$, when $c_t = e'_t$ then c_t is G'_{tgt}.po-maximal and there is no $G'_{\text{tgt}}.\text{po}(c_t, b_t)$. Hence the condition holds between G'_{src} and G'_{tgt}.

(c)

$$\begin{array}{l}\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'. \wedge G'_{\text{tgt}}.\mathsf{po}(c_t, a_t) \implies \\ \exists c_s, a_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge G'_{\text{src}}.\mathsf{po}(c_s, a_s)\end{array}$$

We know this condition holds in G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, and $\mathbb{M}'$, when $c_t = e'_t$ then c_t is G'_{tgt}.po-maximal and there is no $G'_{\text{tgt}}.\text{po}(c_t, a_t)$. Hence the condition holds between G'_{src} and G'_{tgt}.

(d)

$$\begin{array}{l}\forall a_t \in A', b_t \in B'.\ \mathsf{imm}(G'_{\text{tgt}}.\mathsf{po})(b_t, a_t) \implies \\ (\exists c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a', b_s, c_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t) \\ \wedge \mathsf{imm}(G'_{\text{tgt}}.\mathsf{po})(c_t, b_t) \wedge \mathsf{imm}(G'_{\text{src}}.\mathsf{po})(c_s, a_s) \wedge \mathsf{imm}(G'_{\text{src}}.\mathsf{po})(a_s, b') \\ \wedge G'_{\text{src}}.\mathsf{cf}(a_s, a') \wedge \mathsf{imm}(G'_{\text{src}}.\mathsf{po})(a', b_s) \wedge b_s.\mathsf{loc} = b'.\mathsf{loc} \wedge b_s.\mathsf{ord} = b'.\mathsf{ord} \\ \wedge G'_{\text{src}}.\mathsf{ew}(b_s, b'))\end{array}$$

We know this condition holds in G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, $e'_t \notin (A' \cup B')$. As a result, this condition holds between G'_{src} and G'_{tgt}.

(e)

$$\begin{array}{l}\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\text{tgt}}.\mathsf{po}(b_t, c_t) \implies \\ \exists b_s, b'', c_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(b_s, b_t) \wedge \mathbb{M}'(b'', b_t) \\ \wedge G'_{\text{src}}.\mathsf{ew}(b_s, b'') \wedge (G'_{\text{src}}.\mathsf{po}(b_s, c_s) \vee G'_{\text{src}}.\mathsf{po}(b'', c_s))\end{array}$$

We know this condition holds in G_{src} and G_{tgt}.
We consider two cases for e_t.
case $e_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B')$:
In this case there exists b_t such that $G_{\text{tgt}}.\text{po}(b_t, e_t)$.
Hence $G'_{\text{tgt}}.\text{po}(e, e'_t)$ implies $G_{\text{tgt}}.\text{po}(b_t, e'_t)$ and the condition holds.
case $e_t \in A'$:
In this case there exists an event e''_s such that $\mathsf{imm}(G'_{\text{src}}.\mathsf{po})(e_s, e''_s)$ where $\mathbb{M}'(e''_s, b_t)$ and $b_t \in B'$ and $\mathsf{imm}(G'_{\text{tgt}}.\mathsf{po})(b_t, e_t)$. Thus the condition holds between G'_{src} and G'_{tgt}.

(f)

$$\begin{array}{l}\forall a_t \in A', b_t \in B'.\ G'_{\text{tgt}}.\mathsf{po}(b_t, a_t) \implies \\ \exists a_s, b_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t) \wedge \neg G'_{\text{src}}.\mathsf{po}(b_s, a_s)\end{array}$$

We know this condition holds in G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, $e'_t \notin (A' \cup B')$. As a result, this condition holds between G'_{src} and G'_{tgt}.

(g)

$$\begin{array}{l}\forall c_t, c'_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\text{tgt}}.\mathsf{po}(c_t, c'_t) \implies \\ \exists c_s, c'_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \wedge \mathbb{M}'(c'_s, c'_t) \wedge G'_{\text{src}}.\mathsf{po}(c_s, c'_s)\end{array}$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, this condition holds between G'_{src} and G'_{tgt} where $c'_t = e'_t$.

(h)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\mathrm{tgt}}.\mathsf{jf}(c_t, a_t) \implies \\ \exists c_s, a_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(c_s, c_t) \land G'_{\mathrm{src}}.\mathsf{jf}(c_s, a_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} for $c_t = e'_t$ where there is no outgoing $G'_{\mathrm{tgt}}.\mathsf{jf}$ edge from e'_t.

(i)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\mathrm{tgt}}.\mathsf{jf}(a_t, c_t) \implies \\ \exists c_s, a_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(c_s, c_t) \land G'_{\mathrm{src}}.\mathsf{jf}(a_s, c_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} for $c_t = e'_t$.

(j)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{jf}(c_t, b_t) \implies \\ \exists b_s, c_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(b_s, b_t) \land \mathbb{M}'(c_s, c_t) \land G'_{\mathrm{src}}.\mathsf{jf}(c_s, b_s) \\ \land(\exists b' \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(b', b_t) \land G'_{\mathrm{src}}.\mathsf{ew}(b_s, b') \implies G'_{\mathrm{src}}.\mathsf{jf}(c_s, b'))$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} for $c_t = e'_t$ where there is no outgoing $G'_{\mathrm{tgt}}.\mathsf{jf}$ edge from e'_t.

(k)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{jf}(b_t, c_t) \implies \\ ((\exists b_s, c_s \in G'_{\mathrm{src}}.\mathsf{E}.\ (\mathbb{M}'(b_s, b_t) \land \nexists b' \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}(b', b_t) \land G'_{\mathrm{src}}.\mathsf{ew}(b_s, b')) \implies \\ G'_{\mathrm{src}}.\mathsf{jf}(b_s, c_s)) \\ \land(\exists b', b_s, c_s \in G'_{\mathrm{src}}.\mathsf{E}.\ (\mathbb{M}'(b_s, b_t) \land \mathbb{M}'(b', b_t) \land \mathbb{M}'(c_s, c_t) \land G'_{\mathrm{src}}.\mathsf{ew}(b_s, b')) \implies \\ G'_{\mathrm{src}}.\mathsf{jf}(b', c_s)))$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, the condition holds between G'_{src} and G'_{tgt} for $c_t = e'_t$.

(l)

$$\forall c_t, c'_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\mathrm{tgt}}.\mathsf{jf}(c_t, c'_t) \implies \\ \exists c_s, c'_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(c'_s, c'_t) \land G'_{\mathrm{src}}.\mathsf{jf}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, (1) this condition holds between G'_{src} and G'_{tgt} where $c'_t = e'_t$. (2) the condition also holds when $c_t = e'_t$ as in that case there is no outgoing edge from e'_t.

(m)

$$\forall c_t \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A', b_t \in B'.\ G'_{\mathrm{tgt}}.\mathsf{mo}(c_t, a_t) \implies \\ \exists c_s, a_s \in G'_{\mathrm{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(a_s, a_t) \land G'_{\mathrm{src}}.\mathsf{mo}(c_s, a_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, for $c_t = e'_t$ the condition holds between G'_{src} and G'_{tgt}.

(n)

$$\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), a_t \in A'.\ G'_{\text{tgt}}.\mathsf{mo}(a_t, c_t) \implies$$
$$\exists c_s, a_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \land \mathbb{M}'(a_s, a_t) \land G'_{\text{src}}.\mathsf{mo}(a_s, c_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, for $c_t = e'_t$ the condition holds between G'_{src} and G'_{tgt}.

(o)

$$\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\text{tgt}}.\mathsf{mo}(c_t, b_t) \implies$$
$$\exists c_s, b_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}(c_s, c_t) \land \mathbb{M}'(b_s, b_t) \land G'_{\text{src}}.\mathsf{mo}(c_s, b_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, for $c_t = e'_t$ the condition holds between G'_{src} and G'_{tgt}.

(p)

$$\forall c_t \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B'), b_t \in B'.\ G'_{\text{tgt}}.\mathsf{mo}(b_t, c_t) \implies$$
$$\exists c_s, b_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(b_s, b_t) \land G'_{\text{src}}.\mathsf{mo}(b_s, c_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, for $c_t = e'_t$ the condition holds between G'_{src} and G'_{tgt}.

(q)

$$\forall c, c' \in G'_{\text{tgt}}.\mathsf{E} \setminus (A' \cup B').\ G'_{\text{tgt}}.\mathsf{mo}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(c'_s, c'_t) \land G'_{\text{src}}.\mathsf{mo}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Considering the definitions of G'_{src}, G'_{tgt}, $\mathbb{M}'$, for $c_t = e'_t$ or $c'_t = e'_t$ the condition holds between G'_{src} and G'_{tgt}.

(r)

$$\forall o_s \in G'_{\text{src}}.\mathcal{W}.\ (\nexists o_t \in G'_{\text{tgt}}.\mathsf{E}.\ \mathbb{M}'(o_s, o_t)) \implies$$
$$\nexists o'_s \in G'_{\text{src}}.\mathsf{E}.\ G'_{\text{src}}.\mathsf{mo}(o_s, o'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src} and G'_{tgt}, $\mathbb{M}'$, $\mathbb{M}'(e'_s, e_t)$ holds. Hence the condition holds between G'_{src} and G'_{tgt}.

(s)

$$\forall c_t, c'_t \in G'_{\text{tgt}}.\mathsf{E}.\ G'_{\text{tgt}}.\mathsf{ew}(c_t, c'_t) \implies$$
$$\exists c_s, c'_s \in G'_{\text{src}}.\mathsf{E}.\ \mathbb{M}'(c_s, c_t) \land \mathbb{M}'(c'_s, c'_t) \land G_{\text{src}}.\mathsf{ew}(c_s, c'_s)$$

We know the condition holds between G_{src} and G_{tgt}. Following the definitions of G'_{src} and G'_{tgt}, $\mathbb{M}'$ the condition holds between G'_{src} and G'_{tgt} for $c_t = e'_t$ or $c'_t = e'_t$.

Hence the invariant holds between G'_{src} and G'_{tgt}.

(3) Condition to show:
there exists pc' *such that*
$\mathsf{X}'_s.\mathsf{E} = \mathbb{S}'$
$\mathsf{X}'_s.\mathsf{po} = G'_{\text{src}}.\mathsf{po} \cap (\mathbb{S}' \times \mathbb{S}')$
$\mathsf{X}'_s.\mathsf{rf} = G'_{\text{src}}.\mathsf{rf} \cap (\mathbb{S}' \times \mathbb{S}')$
$\mathsf{X}'_s.\mathsf{mo} = G'_{\text{src}}.\mathsf{mo} \cap (\mathbb{S}' \times \mathbb{S}')$
where $\mathbb{S}'(G'_{\text{src}}, \mathsf{pc}') \triangleq \{e \mid e \in G'_{\text{src}}.\mathsf{E} \land G'_{\text{src}}.\mathsf{po}^?(e, \mathsf{pc}'(e.\mathsf{tid}))\}$.

If $e'_t \notin \mathsf{X}'_t$ then $\mathsf{X}'_t = \mathsf{X}_t$. In this case $\mathsf{pc}' = \mathsf{pc}$, $\mathbb{S}' = \mathbb{S}$, and $\mathsf{X}'_s = \mathsf{X}_s$.

Otherwise, when $e'_t \in X'_t$ then X'_t is an extension of X_t, that is,

$$\begin{aligned}
X'_t.\mathsf{E} =& X_t.\mathsf{E} \uplus \{e'_t\}\\
X'_t.\mathsf{po} =& (X_t.\mathsf{po} \uplus \{(a, e'_t) \mid a \in X_t.\mathsf{E} \wedge G'_{\mathrm{tgt}}.\mathsf{po}(a, e'_t)\} \uplus \{(e_t, e'_t)\})^+\\
X'_t.\mathsf{rf} =& X_t.\mathsf{rf} \uplus \{(a, e'_t) \mid a \in X_t.\mathsf{E} \wedge G'_{\mathrm{tgt}}.\mathsf{rf}(a, e'_t)\}\\
&\uplus \{(e'_t, a) \mid a \in X_t.\mathsf{E} \wedge G'_{\mathrm{tgt}}.\mathsf{rf}(e'_t, a)\}\\
X'_t.\mathsf{mo} =& X_t.\mathsf{mo} \uplus \{(a, e'_t) \mid a \in X_t.\mathsf{E} \wedge G'_{\mathrm{tgt}}.\mathsf{mo}(a, e'_t)\}\\
&\uplus \{(e'_t, a) \mid a \in X_t.\mathsf{E} \wedge G'_{\mathrm{tgt}}.\mathsf{mo}(e'_t, a)\}
\end{aligned}$$

We also know that the X_t and X_s are related as follows.
$X_s.\mathsf{E} = X_t.\mathsf{E}$
$X_s.\mathsf{po} = \{(a_s, b_s) \mid \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge X_t.\mathsf{po}(a_t, b_t) \wedge \neg(a_t \in A \wedge b_t \in B)\}$
$\cup \{(a_s, b_s) \mid \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge X_t.\mathsf{po}(b_t, a_t) \wedge (a_t \in A \wedge b_t \in B)\}$
$X_s.\mathsf{rf} = \{(a_s, b_s) \mid \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge X_t.\mathsf{rf}(a_t, b_t)\}$
$X_s.\mathsf{mo} = \{(a_s, b_s) \mid \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge X_t.\mathsf{mo}(a_t, b_t)\}$

Source Execution Extraction.
From X'_t we derive X'_s and relate X'_s to X_s
$X'_s.\mathsf{E} = X'_t.\mathsf{E} = X_t.\mathsf{E} \uplus \{e_t, e'_t\} = X_s.\mathsf{E} \uplus \{e_t, e'_t\}$
$X'_s.\mathsf{po} = \{(a_s, b_s) \mid X'_t.\mathsf{po}(a_t, b_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t)$
$\wedge \neg(a_t \in A' \wedge b_t \in B')\}$
$\cup \{(a_s, b_s) \mid \mathbb{M}(a_s, a_t) \wedge \mathbb{M}(b_s, b_t) \wedge X'_t.\mathsf{po}(b_t, a_t) \wedge (a_t \in A' \wedge b_t \in B')\}$
$\implies X'_s.\mathsf{po} = X_s.\mathsf{po} \cup \{(a_s, e'_s) \mid X'_t.\mathsf{po}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$

$X'_s.\mathsf{rf} = \{(a_s, b_s) \mid X'_t.\mathsf{rf}(a_t, b_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t)\}$
$\implies X'_s.\mathsf{rf} = \{(a_s, b_s) \mid X_t.\mathsf{rf}(a_t, b_t) \wedge \mathbb{M}'(a_s, b_s) \wedge \mathbb{M}'(b_s, b_t)\}$
$\cup \{(a_s, e'_s) \mid X'_t.\mathsf{rf}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(e'_s, a_s) \mid X'_t.\mathsf{rf}(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\implies X'_s.\mathsf{rf} = X_s.\mathsf{rf}$
$\cup \{(a_s, e'_s) \mid X'_t.\mathsf{rf}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(e'_s, a_s) \mid X'_t.\mathsf{rf}(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$

$X'_s.\mathsf{mo} = \{(a_s, b_s) \mid X'_t.\mathsf{mo}(a_t, b_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(b_s, b_t)\}$
$\implies X'_s.\mathsf{mo} = \{(a_s, b_s) \mid X_t.\mathsf{mo}(a_t, b_t) \wedge \mathbb{M}'(a_s, b_s) \wedge \mathbb{M}'(b_s, b_t)\}$
$\cup \{(a_s, e'_s) \mid X'_t.\mathsf{mo}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(e'_s, a_s) \mid X'_t.\mathsf{mo}(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$
$\implies X'_s.\mathsf{mo} = X_s.\mathsf{mo}$
$\cup \{(a_s, e'_s) \mid X'_t.\mathsf{mo}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(e'_s, a_s) \mid X'_t.\mathsf{mo}(e'_t, a_t) \wedge \mathbb{M}'(e'_s, e'_t) \wedge \mathbb{M}'(a_s, a_t)\}$

In this case $\mathrm{pc}' = \mathrm{pc}[e'_s.\mathrm{tid} \mapsto e'_s]$ and hence $\mathbb{S}' = \mathbb{S} \uplus \{e'_s\}$.
Now we relate X'_s and $\mathbb{S}'$.
$X'_s.\mathsf{E} = X_s.\mathsf{E} \uplus \{e'_s\} = \mathbb{S} \uplus \{e'_s\} = \mathbb{S}'$

We already have
$X'_s.\mathsf{po} = (X_s.\mathsf{po} \cup \{(a_s, e'_s) \mid X'_t.\mathsf{po}(a_t, e'_t) \wedge \mathbb{M}'(a_s, a_t) \wedge \mathbb{M}'(e'_s, e'_t)\})^+$
$\implies X'_s.\mathsf{po} = G_{\mathrm{src}}.\mathsf{po} \cap (\mathbb{S} \times \mathbb{S}) \cup \{G'_{\mathrm{src}}.\mathsf{po}(a_s, e'_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\implies X'_s.\mathsf{po} = G'_{\mathrm{src}}.\mathsf{po} \cap (\mathbb{S}' \times \mathbb{S}')$

We already have

$\mathsf{X}'_s.\mathsf{rf} = \mathsf{X}_s.\mathsf{rf}$
$\cup \{(a_s, e'_s) \mid \mathsf{X}'_t.\mathsf{rf}(a_t, e'_t) \land \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(e'_s, a_s) \mid \mathsf{X}'_t.\mathsf{rf}(e'_t, a_t) \land \mathbb{M}'(e'_s, e'_t) \land \mathbb{M}'(a_s, a_t)\}$
$\implies \mathsf{X}'_s.\mathsf{rf} = G_{\mathsf{src}}.\mathsf{rf} \cap (\mathbb{S} \times \mathbb{S}) \cup \{G'_{\mathsf{src}}.\mathsf{rf}(a_s, e'_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\cup \{G'_{\mathsf{src}}.\mathsf{rf}(e'_s, a_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\implies \mathsf{X}'_s.\mathsf{rf} = G'_{\mathsf{src}}.\mathsf{rf} \cap (\mathbb{S}' \times \mathbb{S}')$
We already have
$\mathsf{X}'_s.\mathsf{mo} = \mathsf{X}_s.\mathsf{mo}$
$\cup \{(a_s, e'_s) \mid \mathsf{X}'_t.\mathsf{mo}(a_t, e'_t) \land \mathbb{M}'(a_s, a_t) \land \mathbb{M}'(e'_s, e'_t)\}$
$\cup \{(e'_s, a_s) \mid \mathsf{X}'_t.\mathsf{mo}(e'_t, a_t) \land \mathbb{M}'(e'_s, e'_t) \land \mathbb{M}'(a_s, a_t)\}$
$\implies \mathsf{X}'_s.\mathsf{mo} = G_{\mathsf{src}}.\mathsf{mo} \cap (\mathbb{S} \times \mathbb{S}) \cup \{G'_{\mathsf{src}}.\mathsf{mo}(a_s, e'_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\cup \{G'_{\mathsf{src}}.\mathsf{mo}(e'_s, a_s) \mid a_s, e_s \in \mathbb{S}'\}$
$\implies \mathsf{X}'_s.\mathsf{mo} = G'_{\mathsf{src}}.\mathsf{mo} \cap (\mathbb{S}' \times \mathbb{S}')$

As a result, $G'_{\mathsf{src}} \sim G'_{\mathsf{tgt}}$.

Thus we complete the construction of the source event structure G_{src} and the source execution X_s can be extracted from G_{src}, that is, $\mathsf{X}_s \in \mathsf{ex}_{\mathrm{WEAKESTMO}}(G_{\mathsf{src}})$.

□

G PROOFS OF CORRECTNESS OF ELIMINATIONS

We restate the definition of compilation correctness and the safe elimination theorem.

Definition 7. A transformation of program $\mathbb{P}_{\mathsf{src}}$ in memory model M_{src} to program $\mathbb{P}_{\mathsf{tgt}}$ in model M_{tgt} is *correct* if it does not introduce new behaviors: i.e., $\mathsf{Behavior}_{M_{\mathsf{tgt}}}(\mathbb{P}_{\mathsf{tgt}}) \subseteq \mathsf{Behavior}_{M_{\mathsf{src}}}(\mathbb{P}_{\mathsf{src}})$.

Theorem 7. *The eliminations in Fig. 12 are correct in both* WEAKESTMO *models.*

The safe eliminations from Fig. 12 are

Definition 10. $\mathsf{elim}(\mathbb{P}_{\mathsf{src}}, \mathbb{P}_{\mathsf{tgt}})$
such that $\mathbb{P}_{\mathsf{tgt}}(i) \subseteq \mathbb{P}_{\mathsf{src}}(i) \cup \{\tau{\cdot}\tau' \mid \tau{\cdot}\alpha{\cdot}\tau' \in \mathbb{P}_{\mathsf{src}}(i)\} \land \forall j \neq i.\ \mathbb{P}_{\mathsf{tgt}}(j) = \mathbb{P}_{\mathsf{src}}(j)$
where α is a label of shared memory accesses or fences..

Then The formal statement is as follows:

$$\begin{array}{l}
\forall \mathbb{P}_{\mathsf{src}}.\ \mathsf{elim}(\mathbb{P}_{\mathsf{src}}, \mathbb{P}_{\mathsf{tgt}}) \implies \\
\quad \forall G_{\mathsf{tgt}}.\ G_{\mathsf{init}} \rightarrow_{\mathbb{P}_{\mathsf{tgt}},\textsc{weakestmo}}{}^{*}\ G_{\mathsf{tgt}}.\ \exists G_{\mathsf{src}}.\ G_{\mathsf{init}} \rightarrow_{\mathbb{P}_{\mathsf{src}},\textsc{weakestmo}}{}^{*}\ G_{\mathsf{src}} \land \\
\quad\quad \forall \mathsf{X}_t \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\mathsf{tgt}}).\ \exists \mathsf{X}_s \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\mathsf{src}}).\ \mathsf{Behavior}(\mathsf{X}_t) = \mathsf{Behavior}(\mathsf{X}_s) \\
\quad\quad \land \mathsf{X}_t.\mathsf{Race} \cap \mathcal{E}_{\textsc{na}} \neq \emptyset \implies \mathsf{X}_s.\mathsf{Race} \cap \mathcal{E}_{\textsc{na}} \neq \emptyset
\end{array}$$

To prove the theorem, we construct a source event structure following a given target event structure. Then, for an extracted consistent target execution we extract a source execution from the source event structure. Then we show that the source execution is consistent and source and target execution has same behavior. Finally, we show race preservation: if target is racy, then the source execution is also racy. As a result, if the target execution has undefined behavior due to a data race, so does the source execution.

Now we study various safe eliminations.

G.1 Overwritten Write (OW)

Proof. Recall the relationship between the two programs for the thread i affected by the transformation:

$$\mathbb{P}_{\mathsf{tgt}}(i) \subseteq \mathbb{P}_{\mathsf{src}}(i) \cup \{\tau{\cdot}\mathsf{St}_o(x,v){\cdot}\tau' \mid \tau{\cdot}\mathsf{St}_{o'}(x,v'){\cdot}\mathsf{St}_o(x,v){\cdot}\tau' \in \mathbb{P}_{\mathsf{src}}(i) \land o' \sqsubseteq o\}$$

For all other threads $j \neq i$, we have $\mathbb{P}_{\mathsf{tgt}}(j) = \mathbb{P}_{\mathsf{src}}(j)$. Assume we have a target event structure, G_{tgt}, and an execution, $\mathsf{X}_t \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\mathsf{tgt}})$, extracted from it.

Let W be the set of stores of thread i of G_{tgt} with label $\mathsf{St}_o(x,v)$, and whose po-prefix has some sequence of labels τ such that $\tau{\cdot}\mathsf{St}_o(x,v) \notin \mathbb{P}_{\mathsf{src}}(i)$. Then, because of the relationship between the two programs, we know that for each such $w \in W$, $\tau{\cdot}\mathsf{St}_{o'}(x,v'){\cdot}\mathsf{St}_o(x,v) \in \mathbb{P}_{\mathsf{src}}(i)$ for the appropriate τ. Let C be the immediate $G_{\mathsf{tgt}}.\mathsf{po}$-predecessors of the events in W.

Source Event Structure Construction. To construct G_{src}, we follow the construction steps of G_{tgt}. For each target construction step that adds event e to G_{tgt} to get G'_{tgt}, we perform one or more corresponding steps going from G_{src} to G'_{src}. We do a case analysis on the event e of the target event structure.

Case $e \notin W$**:** In this case, we append event e to the source event structure as follows:

$$\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{e\} \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(a,e) \mid a \in \mathsf{dom}(G'_{\mathsf{tgt}}.\mathsf{po};[e])\})^{+} \\
G'_{\mathsf{src}}.\mathsf{jf} &= G'_{\mathsf{tgt}}.\mathsf{jf} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G'_{\mathsf{tgt}}.\mathsf{mo} \cup \mathsf{imm}(G_{\mathsf{src}}.\mathsf{po});[W];G'_{\mathsf{tgt}}.\mathsf{mo} \cup G'_{\mathsf{tgt}}.\mathsf{mo};[W];\mathsf{imm}(G_{\mathsf{src}}.\mathsf{po}^{-1}) \\
G'_{\mathsf{src}}.\mathsf{ew} &= G'_{\mathsf{tgt}}.\mathsf{ew}
\end{aligned}$$

Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} are consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

From the definition, there is no $G_{\mathsf{src}}.\mathsf{hb}; G_{\mathsf{src}}.\mathsf{eco}^?$ as well as $G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^?$ cycle. Compared to G_{src} and G'_{tgt}, the additional $G'_{\mathsf{src}}.\mathsf{mo}$ edges are from and to events the deleted events.

Let $d \in (G'_{\mathsf{src}}.\mathsf{E} \setminus G'_{\mathsf{tgt}}.\mathsf{E})$ be such a deleted event. Assume the mo edges to or from d creates a $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle. However, for each $G'_{\mathsf{src}}.\mathsf{mo}(d, e)$ or $G'_{\mathsf{src}}.\mathsf{mo}(e, d)$ already there exists $G'_{\mathsf{src}}.\mathsf{mo}(w, e)$ or $G'_{\mathsf{src}}.\mathsf{mo}(e, w)$ respectively where $w \in W$ and $\mathsf{imm}(G_{\mathsf{src}}.\mathsf{po}(d, w))$. Thus event e results no new $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle and hence G'_{src} satisfies (COH′).

Case $e \in W$: In this case, we first append a new event d with $d.\mathsf{lab} = \mathsf{St}_{o'}(x, v')$ and then the event e to G_{src} as follows:

$$\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{d, e\} \quad \text{where } d.\mathsf{lab} = \mathsf{St}_{o'}(x, v')\\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(d, e)\} \uplus \{(c, d) \mid (c, e) \in G'_{\mathsf{tgt}}.\mathsf{po}\})^+\\
G'_{\mathsf{src}}.\mathsf{jf} &= G'_{\mathsf{tgt}}.\mathsf{jf}\\
G'_{\mathsf{src}}.\mathsf{mo} &= G'_{\mathsf{tgt}}.\mathsf{mo} \uplus \{(d, a) \mid G'_{\mathsf{tgt}}.\mathsf{mo}(e, a)\} \uplus \{(a, d) \mid G'_{\mathsf{tgt}}.\mathsf{mo}(a, e)\} \uplus \{(d, e)\}\\
G'_{\mathsf{src}}.\mathsf{ew} &= G'_{\mathsf{tgt}}.\mathsf{ew}
\end{aligned}$$

Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

From the definition, there is no $G_{\mathsf{src}}.\mathsf{hb}; G_{\mathsf{src}}.\mathsf{eco}^?$ as well as $G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^?$ cycle. Compared to G_{src} and G'_{tgt}, the additional $G'_{\mathsf{src}}.\mathsf{mo}$ edges are from and to the event d. Assume the mo edges to or from d creates a $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle.

However, for each $G'_{\mathsf{src}}.\mathsf{mo}(d, a)$ or $G'_{\mathsf{src}}.\mathsf{mo}(a, d)$ already there exists $G'_{\mathsf{src}}.\mathsf{mo}(w, e)$ or $G'_{\mathsf{src}}.\mathsf{mo}(e, w)$ respectively where $a \neq e$. Thus event e results no new $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle and hence G'_{src} satisfies (COH′).

Source Execution Construction. Next, we construct an execution $\mathsf{X}_t \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{tgt}})$.

If $W \subseteq (G_{\mathsf{tgt}}.\mathsf{E} \setminus \mathsf{X}_t.\mathsf{E})$, then we find the corresponding execution $\mathsf{X}_s \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{src}})$ such that X_s contains no event created for $\mathsf{St}_{o'}(x, v')$. Else if an event $w_t \in W$ is in X_t, then we know that we can find an execution with $w_s \in \mathsf{X}_s.\mathsf{E}$ and $\mathsf{X}_s.\mathsf{E}$ also contains an event w' corresponding to $\mathsf{store}_{o'}(x, v')$. Thus X_s is as follows.

$$\begin{aligned}
\mathsf{X}_s.\mathsf{E} &= \mathsf{X}_t.\mathsf{E} \uplus \{d \mid \mathsf{X}_t.\mathsf{E} \cap W \neq \emptyset\}\\
\mathsf{X}_s.\mathsf{po} &= (\mathsf{X}_t.\mathsf{po} \uplus \{(c, d), (d, w) \mid (c, w) \in \mathsf{imm}(\mathsf{X}_t.\mathsf{po}) \cap (C \times W) \wedge d \in (G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E})\})^+\\
\mathsf{X}_s.\mathsf{rf} &= \mathsf{X}_t.\mathsf{rf}\\
\mathsf{X}_s.\mathsf{mo} &= \mathsf{X}_t.\mathsf{mo} \uplus \{(d, w) \mid (d, w) \in ((G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E}) \times W)\}\\
&\quad \uplus \{(a, d) \mid \mathsf{X}_t.\mathsf{mo}(a, w) \wedge (d, w) \in ((G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E}) \times W) \cap \mathsf{imm}(G_{\mathsf{src}}.\mathsf{po})\}\\
&\quad \uplus \{(d, a) \mid \mathsf{X}_t.\mathsf{mo}(w, a) \wedge (d, w) \in ((G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E}) \times W) \cap \mathsf{imm}(G_{\mathsf{src}}.\mathsf{po})\}
\end{aligned}$$

Source Execution Consistency. Now we check the consistency of X_s.

Since X_t is consistent, the (Well-formed), (total-MO), (Coherence), (Atomicity) constraints also hold for X_s. The (SC) constraint is affected only when $o = o' = \mathsf{sc}$, in which case the new events introduce some $[\mathsf{SC}], \mathsf{X}_s.\mathsf{po}_x; [\mathsf{SC}]$ edges. These edges, however, can create a $(\mathsf{X}_s.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_s.\mathsf{psc}_{\mathsf{F}})$

cycle only when there is a $(\mathsf{X}_t.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_t.\mathsf{psc}_{\mathsf{F}})$ cycle. Since X_t is consistent there is no $(\mathsf{X}_t.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_t.\mathsf{psc}_{\mathsf{F}})$ cycle. Hence, X_s satisfies (SC) and, as a result, X_s is consistent.

Same Behavior. For locations $y \neq x$, we have $\mathsf{X}_s.\mathsf{E}_y = \mathsf{X}.\mathsf{E}_y$ and as a result $\mathsf{Behavior}(\mathsf{X}_s)|_y = \mathsf{Behavior}(\mathsf{X}_t)|_y$ trivially holds. Now we check whether $\mathsf{Behavior}(\mathsf{X}_s)|_x = \mathsf{Behavior}(\mathsf{X}_t)|_x$ holds. Note that any newly introduced event $d \in \mathsf{X}_s.\mathsf{E} \setminus \mathsf{X}_t.\mathsf{E}$ is not $\mathsf{X}_s.\mathsf{mo}$ maximal, because in that case there exists $w \in W$ such that $\mathsf{X}_s.\mathsf{mo}(d, w)$. Hence $\mathsf{Behavior}(\mathsf{X}_s) = \mathsf{Behavior}(\mathsf{X}_t)$ holds.

Race Preservation. Moreover, if X_t is racy, then the new write d does not introduce any $\mathsf{X}_s.\mathsf{sw}_{\mathsf{C11}}$ edge in X_s. Hence X_s is also racy. As a result, if the target execution has undefined behavior due to a data race, so does the source execution. □

G.2 Read after Write (RAW)

Proof. Recall the relationship between the two programs for the thread i affected by the transformation:

$$\mathbb{P}_{\mathsf{tgt}}(i) \subseteq \mathbb{P}_{\mathsf{src}}(i) \cup \{\tau \cdot \mathsf{St}_o(x, v) \cdot \tau' \mid \tau \cdot \mathsf{St}_o(x, v) \cdot \mathsf{Ld}_{o'}(x, \_) \cdot \tau' \in \mathbb{P}_{\mathsf{src}}(i) \wedge o' \sqsubseteq o\}$$

or

$$\mathbb{P}_{\mathsf{tgt}}(i) \subseteq \mathbb{P}_{\mathsf{src}}(i) \cup \{\tau \cdot \mathsf{U}_o(x, v', v) \cdot \tau' \mid \tau \cdot \mathsf{U}_o(x, v', v) \cdot \mathsf{Ld}_{o'}(x, \_) \cdot \tau' \in \mathbb{P}_{\mathsf{src}}(i) \wedge o' \sqsubseteq o\}$$

For all other threads $j \neq i$, we have $\mathbb{P}_{\mathsf{tgt}}(j) = \mathbb{P}_{\mathsf{src}}(j)$. Assume we have a target event structure, G_{tgt}, and an execution, $\mathsf{X}_t \in \mathsf{ex}_{\textsc{weakestmo}}(G_{\mathsf{tgt}})$, extracted from it.

Let W be the set of writes with label $\mathsf{St}_o(x, v)$ or $\mathsf{U}_o(x, v', v)$ in the target event structure G_{tgt} for the respective accesses and whose po-suffix has some sequence of labels τ' such that $\mathsf{St}_o(x, v) \cdot \tau' \notin \mathbb{P}_{\mathsf{src}}(i)$ or $\mathsf{U}_o(x, v', v) \cdot \tau' \notin \mathbb{P}_{\mathsf{src}}(i)$ respectively. Then, because of the relationship between the two programs, we know that for each such $w \in W$, $\mathsf{St}_o(x, v) \cdot \mathsf{Ld}_{o'}(x, \_) \cdot \tau' \in \mathbb{P}_{\mathsf{src}}(i)$ or $\mathsf{U}_o(x, v', v) \cdot \mathsf{Ld}_{o'}(x, \_) \cdot \tau' \in \mathbb{P}_{\mathsf{src}}(i)$ respectively for the appropriate τ'. Let C be the immediate $G_{\mathsf{tgt}}.\mathsf{po}$-successors of the events in W.

Source Event Structure Construction.

To construct G_{src}, we follow the construction steps of G_{tgt}. For each target construction step that adds event e to G_{tgt} to get G'_{tgt}, we perform one or more corresponding steps going from G_{src} to G'_{src}. We do a case analysis on the event e of the target event structure.

Case $e \notin W$: In this case we append event e to the source event structure as follows:

$$\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{e\} \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(a, e) \mid a \notin W \wedge \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po})(a, e)\} \\
&\qquad \uplus \{(r, e) \mid w \in W \wedge \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po})(w, e)\})^+ \\
G'_{\mathsf{src}}.\mathsf{jf} &= G_{\mathsf{src}}.\mathsf{jf} \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{jf}(a, e)\} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G'_{\mathsf{tgt}}.\mathsf{mo} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G'_{\mathsf{tgt}}.\mathsf{ew}
\end{aligned}$$

Now we check the consistency of G'_{src} event structure. We already know that G_{src} and G'_{tgt} are consistent.

Following the definition of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ), (COH′) constraints immediately hold and hence G'_{src} is also consistent.

Case $e \in W$**:** In this case we first append event e and then event r with $r.\mathsf{lab} = \mathsf{Ld}_{o'}(x, v)$ to G_{src} as follows:

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{r, e\} \quad \text{where } r.\mathsf{lab} = \mathsf{Ld}_{o'}(x, v) \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(e, r), (a, e) \mid \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po})(a, e)\})^+ \\
G'_{\mathsf{src}}.\mathsf{jf} &= G_{\mathsf{src}}.\mathsf{jf} \uplus \{(e, r)\} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G'_{\mathsf{tgt}}.\mathsf{mo} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G'_{\mathsf{tgt}}.\mathsf{ew}
\end{aligned}
$$

Now we check the consistency of G'_{src}.

We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′). The outgoing edges from r are $G'_{\mathsf{src}}.\mathsf{fr}$. Hence for an outgoing edge $G'_{\mathsf{src}}.\mathsf{fr}(r, a)$, there is $G_{\mathsf{src}}.\mathsf{mo}(e, a)$ edge. If $G'_{\mathsf{src}}.\mathsf{fr}(r, a)$ results in a $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle, then $G_{\mathsf{src}}.\mathsf{hb}; G_{\mathsf{src}}.\mathsf{eco}^?$ cycle is already there in G_{src}. But we know that G_{src} is consistent and hence $G_{\mathsf{src}}.\mathsf{hb}; G_{\mathsf{src}}.\mathsf{eco}^?$ is not possible. Hence a contradiction and $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ is also not possible. Thus G'_{src} preserves (COH′) and G'_{src} is consistent.

Source Execution Construction. Next, we construct an execution $\mathsf{X}_t \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{tgt}})$.

If $W \subseteq (G_{\mathsf{tgt}} \setminus \mathsf{X}_t.\mathsf{E})$, then we find the corresponding execution $\mathsf{X}_s \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{src}})$ such that X_s contains no $\mathsf{St}_o(x, v)$ or $\mathsf{U}_o(x, v', v)$. In that case X_s also does not contain any event created for $\mathsf{Ld}_{o'}(x, v)$ access.

Else if an event $w \in W$ is in X_t, then we know that we can find a source execution X_s which contains both w and r. Thus X_s is as follows.

Thus X_s is as follows.

$$
\begin{aligned}
\mathsf{X}_s.\mathsf{E} &= \mathsf{X}_t.\mathsf{E} \uplus \{r \mid \mathsf{X}_t.\mathsf{E} \cap W \neq \emptyset\} \\
\mathsf{X}_s.\mathsf{po} &= (\mathsf{X}_t.\mathsf{po} \uplus \{(w, r), (r, c) \mid (w, c) \in \mathsf{imm}(\mathsf{X}_t.\mathsf{po}) \cap (W \times C) \wedge r \in (G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E})\})^+ \\
\mathsf{X}_s.\mathsf{rf} &= \mathsf{X}_t.\mathsf{rf} \uplus \{(w, r) \mid w \in \mathsf{X}_t.\mathsf{E} \cap W\} \\
\mathsf{X}_s.\mathsf{mo} &= \mathsf{X}_t.\mathsf{mo}
\end{aligned}
$$

Source Execution Consistency. Now we check the consistency of X_s.

We know that X_t is consistent. The (Well-formed), (total-MO), (Coherence), (Atomicity) constraints hold as they hold for X_t. Considering the (SC) constraint we observe that if $o = o' = \text{SC}$, then r' introduces a $[\mathsf{SC}], \mathsf{X}_s.\mathsf{po}_x; [\mathsf{SC}]$ edge. This edge can create a $(\mathsf{X}_s.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_s.\mathsf{psc}_{\mathsf{F}})$ cycle only when there is a $(\mathsf{X}_t.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_t.\mathsf{psc}_{\mathsf{F}})$ cycle. Since X_t is consistent there is no $(\mathsf{X}_t.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_t.\mathsf{psc}_{\mathsf{F}})$ cycle. Hence there is no $(\mathsf{X}_s.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_s.\mathsf{psc}_{\mathsf{F}})$ cycle and X_s satisfies (SC). As a result, X_s is consistent.

Same Behavior.

Now we check whether $\mathsf{Behavior}(\mathsf{X}_s) = \mathsf{Behavior}(\mathsf{X}_t)$ holds.

For locations $y \neq x$, $\mathsf{Behavior}|_y(\mathsf{X}_s) = \mathsf{Behavior}|_y(\mathsf{X}_t)$ holds.

For x load r' does not introduce any new mo edge and hence does not affect behavior of X_s.

Hence $\mathsf{Behavior}(\mathsf{X}_s) = \mathsf{Behavior}(\mathsf{X}_t)$ holds.

Race Preservation.

Moreover, if X_t is racy, then the new read r' does not introduce any new $(\mathsf{X}_s.\mathsf{sw}_{\mathsf{C11}} \setminus \mathsf{X}_s.\mathsf{po})$ edge in X_s. Hence X_s is also racy. As a result, if the target execution has undefined behavior due to data race then the source execution also has undefined behavior due to data race.

□

G.3 Read after Read (RAR)

Proof. Recall the relationship between the two programs for the thread i affected by the transformation:

$$\mathbb{P}_{\mathsf{tgt}}(i) \subseteq \mathbb{P}_{\mathsf{src}}(i) \cup \{\tau\cdot\mathsf{Ld}_o(x,v)\cdot\tau' \mid \tau\cdot\mathsf{Ld}_o(x,v)\cdot\mathsf{Ld}_{o'}(x,\_)\cdot\tau' \in \mathbb{P}_{\mathsf{src}}(i) \land o' \sqsubseteq o\}$$

For all other threads $j \neq i$, we have $\mathbb{P}_{\mathsf{tgt}}(j) = \mathbb{P}_{\mathsf{src}}(j)$. Assume we have a target event structure, G_{tgt}, and an execution, $\mathsf{X}_t \in \mathsf{ex}_{\mathsf{WEAKESTMO}}(G_{\mathsf{tgt}})$, extracted from it.

Let R be the set of loads with label $\mathsf{Ld}_o(x,v)$ in the target event structure G_{tgt} whose po-suffix has some sequence of labels τ' such that $\mathsf{Ld}_o(x,v)\cdot\tau' \notin \mathbb{P}_{\mathsf{src}}(i)$. Then, because of the relationship between the two programs, we know that for each such $r \in W$, for the appropriate τ', $\mathsf{Ld}_o(x,v)\cdot\mathsf{Ld}_{o'}(x,\_)\cdot\tau' \in \mathbb{P}_{\mathsf{src}}(i)$ holds. Let C be the immediate G_{tgt}.po-successors of the events in R.

Source Event Structure Construction.

To construct G_{src}, we follow the construction steps of G_{tgt}. For each target construction step that adds event e to G_{tgt} to get G'_{tgt}, we perform one or more corresponding steps going from G_{src} to G'_{src}. We do a case analysis on the event e of the target event structure.

Case $e \notin R$: In this case we append event e to the source event structure as follows:

$$\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{e\} \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(a,e) \mid a \notin R \land \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po})(a,e)\} \\
&\qquad \uplus \{(d,e) \mid r \in R \land \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po})(r,e)\})^+ \\
G'_{\mathsf{src}}.\mathsf{jf} &= G'_{\mathsf{tgt}}.\mathsf{jf} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G'_{\mathsf{tgt}}.\mathsf{mo} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G'_{\mathsf{tgt}}.\mathsf{ew}
\end{aligned}$$

Now we check the consistency of G'_{src} event structure. We already know that G_{src} and G'_{tgt} are consistent.

Following the definition of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ), (COH′) constraints immediately hold and hence G'_{src} is also consistent.

Case $e \in R$: In this case we first append event e and then event r with $r.\mathsf{lab} = \mathsf{Ld}_{o'}(x,v)$ to G_{src} as follows:

$$\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{d,e\} \quad \text{where } d.\mathsf{lab} = \mathsf{Ld}_{o'}(x,v) \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(e,d),(a,e) \mid \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po})(a,e)\})^+ \\
G'_{\mathsf{src}}.\mathsf{jf} &= G_{\mathsf{src}}.\mathsf{jf} \uplus \{(a,e),(a,d) \mid G'_{\mathsf{tgt}}.\mathsf{jf}(a,e)\} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G'_{\mathsf{tgt}}.\mathsf{mo} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G'_{\mathsf{tgt}}.\mathsf{ew}
\end{aligned}$$

Now we check the consistency of G'_{src}.

We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′). The outgoing edges from d are $G'_{\mathsf{src}}.\mathsf{fr}$. Hence for an outgoing edge $G'_{\mathsf{src}}.\mathsf{fr}(d,a)$ there is $G'_{\mathsf{src}}.\mathsf{fr}(e,a)$ as well as $G'_{\mathsf{tgt}}.\mathsf{fr}(e,a)$ edges. Hence if $G'_{\mathsf{src}}.\mathsf{fr}(d,a)$ results in a $G'_{\mathsf{src}}.\mathsf{hb}\ ;\ G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle, then there is also $G'_{\mathsf{tgt}}.\mathsf{hb};G'_{\mathsf{tgt}}.\mathsf{eco}^?$ cycle. But we know that G'_{tgt} is consistent and hence $G'_{\mathsf{tgt}}.\mathsf{hb};G'_{\mathsf{tgt}}.\mathsf{eco}^?$ cycle is not possible. Hence a contradiction and $G'_{\mathsf{src}}.\mathsf{hb};G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle is also not possible. Thus G'_{src} preserves (COH′) and G'_{src} is consistent.

Source Execution Construction. Next, we construct an execution $\mathsf{X}_t \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\text{tgt}})$.

If $R \subseteq (G_{\text{tgt}} \setminus \mathsf{X}_t.\mathsf{E})$, then we find the corresponding execution $\mathsf{X}_s \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\text{src}})$ such that X_s contains no $\mathsf{Ld}_o(x, v)$. In that case X_s also does not contain any event created for $\mathsf{Ld}_{o'}(x, v)$ access.

Else if an event $r \in R$ is in X_t, then we know that we can find a source execution X_s which contains both r and d. Thus X_s is as follows.

Thus X_s is as follows.

$$\mathsf{X}_s.\mathsf{E} = \mathsf{X}_t.\mathsf{E} \uplus \{d \mid \mathsf{X}_t.\mathsf{E} \cap R \neq \emptyset\}$$

$$\mathsf{X}_s.\mathsf{po} = (\mathsf{X}_t.\mathsf{po} \uplus \{(r, d), (d, c) \mid \in (r, c) \in \mathsf{imm}(\mathsf{X}_t.\mathsf{po}) \cap (R \times C) \wedge d \in (G_{\text{src}}.\mathsf{E} \setminus G_{\text{tgt}}.\mathsf{E})\})^+$$

$$\mathsf{X}_s.\mathsf{rf} = \mathsf{X}_t.\mathsf{rf} \uplus \{(a, d) \mid a \in \mathsf{dom}(\mathsf{X}_t.\mathsf{rf}; [R])\}$$

$$\mathsf{X}_s.\mathsf{mo} = \mathsf{X}_t.\mathsf{mo}$$

Source Execution Consistency. Now we check the consistency of X_s.

We know that X_t is consistent. The (Well-formed), (total-MO), (Coherence), (Atomicity) constraints hold as they hold for X_t. Considering the (SC) constraint we observe that if $o = o' = \text{SC}$, then r' introduces a $[\mathsf{SC}], \mathsf{X}_s.\mathsf{po}_x; [\mathsf{SC}]$ edge. This edge can create a $(\mathsf{X}_s.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_s.\mathsf{psc}_{\mathsf{F}})$ cycle only when there is a $(\mathsf{X}_t.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_t.\mathsf{psc}_{\mathsf{F}})$ cycle. Since X_t is consistent there is no $(\mathsf{X}_t.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_t.\mathsf{psc}_{\mathsf{F}})$ cycle. Hence there is no $(\mathsf{X}_s.\mathsf{psc}_{\mathsf{base}} \cup \mathsf{X}_s.\mathsf{psc}_{\mathsf{F}})$ cycle and X_s satisfies (SC). As a result, X_s is consistent.

Same Behavior.

Now we check whether $\mathsf{Behavior}(\mathsf{X}_s) = \mathsf{Behavior}(\mathsf{X}_t)$ holds.

For locations $y \neq x$, $\mathsf{Behavior}|_y(\mathsf{X}_s) = \mathsf{Behavior}|_y(\mathsf{X}_t)$ holds.

For x, load r' does not introduce any new mo edge and hence does not affect behavior of X_s.

Hence $\mathsf{Behavior}(\mathsf{X}_s) = \mathsf{Behavior}(\mathsf{X}_t)$ holds.

Race Preservation.

Moreover, if X_t is racy, then the new read d does not introduce any new $(\mathsf{X}_s.\mathsf{hb}_{\mathsf{C11}} \setminus \mathsf{X}_s.\mathsf{po})$ relation in X_s. Hence X_s is also racy. As a result, if the target execution has undefined behavior due to data race then the source execution also has undefined behavior due to data race. □

G.4 Non-Atomic Read-Write (naRW)

Proof. Recall the relationship between the two programs for the thread i affected by the transformation:

$$\mathbb{P}_{\text{tgt}}(i) \subseteq \mathbb{P}_{\text{src}}(i) \cup \{\tau \cdot \tau' \mid \tau \cdot \mathsf{Ld}_{\text{NA}}(x, v) \cdot \mathsf{St}_{\text{NA}}(x, v) \cdot \tau' \in \mathbb{P}_{\text{src}}(i)\}$$

For all other threads $j \neq i$, we have $\mathbb{P}_{\text{tgt}}(j) = \mathbb{P}_{\text{src}}(j)$. Assume we have a target event structure, G_{tgt}, and an execution, $\mathsf{X}_t \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\text{tgt}})$, extracted from it.

Let C be the set of events the target event structure G_{tgt} whose po-suffix has some sequence of labels τ' such that $c \cdot \tau' \notin \mathbb{P}_{\text{src}}(i)$ where $c \in C$. Also let D be the set of events which are immediate po-successors of events in C. Then, because of the relationship between the two programs, we know that for each such $c \in C$ and $c \in \tau$, $c \cdot \mathsf{Ld}_{\text{NA}}(x, v) \cdot \mathsf{St}_{\text{NA}}(x, v) \cdot \tau' \in \mathbb{P}_{\text{src}}(i)$ for the appropriate τ'.

Source Event Structure Construction.

To construct G_{src}, we follow the construction steps of G_{tgt}. For each target construction step that adds event e to G_{tgt} to get G'_{tgt}, we perform one or more corresponding steps going from G_{src} to G'_{src}. We do a case analysis on the event e of the target event structure.

Case $e \in C$**:** In this case we append event e followed by $\mathsf{Ld}_{\mathrm{NA}}(x, s.\mathsf{wval})$ justified from a write s and $\mathsf{St}_{\mathrm{NA}}(x, s.\mathsf{wval})$ to the source event structure as follows:

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}} \uplus \{e, r, w\} \quad \text{where } r.\mathsf{lab} = \mathsf{Ld}_{\mathrm{NA}}(x, \_) \text{ and } w = \mathsf{St}_{\mathrm{NA}}(x, \_) \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(a, e), (e, r), (r, w) \mid G_{\mathsf{tgt}}.\mathsf{po}(a, e)\})^{+} \\
G'_{\mathsf{src}}.\mathsf{jf} &= G_{\mathsf{src}}.\mathsf{jf} \uplus \{(a, e), (s, r) \mid G'_{\mathsf{tgt}}.\mathsf{jf}(a, e) \land \mathsf{existsW}(G'_{\mathsf{src}}, s, r)\} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G_{\mathsf{src}}.\mathsf{mo} \uplus \{(a, w) \mid a \in (G_{\mathsf{src}}.\mathcal{W}_x \setminus \mathsf{WA})\} \uplus \{(w, a) \mid a \in \mathsf{WA}\} \\
&\qquad \text{where } \mathsf{WA} = \{a \mid (G'_{\mathsf{tgt}}.\mathsf{ew}^?; G'_{\mathsf{tgt}}.\mathsf{mo})(s, a)\} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G_{\mathsf{src}}.\mathsf{ew} \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{ew}(a, e)\}
\end{aligned}
$$

Now we check the consistency of G'_{src}.

We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src} and considering the definition of Remark 3, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′). Again following the Remark 3 definition, additional events r and w do not create any $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle. Hence G'_{src} satisfies (COH′) and is consistent. **Case** $e \notin C$**:** In this case we append event e to the source event structure. However, if

e is justified-from s in G'_{tgt} and happens-after the newly newly appended non-atomic store from $(G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E})$ in G'_{src}, then e is justified-from the new store $\mathsf{St}_{\mathrm{NA}}(X, s.\mathsf{wval})$. Let $W \subseteq (G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E})$ be the set of such store events. Note that id event e happens-after event $w \in W$, then there exists an intermediate event $d \in D$. Thus we construct G'_{src} as follows:

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{e\} \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{po}(a, e)\} \\
&\qquad \uplus \{(w, e) \mid w \in W \land e \in \mathsf{codom}([C]; \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po}); [D])\})^{+} \\
G'_{\mathsf{src}}.\mathsf{jf} &= G_{\mathsf{src}}.\mathsf{jf} \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{jf}(a, e) \land e \notin \mathsf{codom}([D]; G_{\mathsf{src}}.\mathsf{hb})\} \\
&\qquad \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{jf}(a, e) \land e \in \mathsf{codom}([D]; G_{\mathsf{src}}.\mathsf{hb})\} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G_{\mathsf{src}}.\mathsf{mo} \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{mo}(a, e)\} \uplus \{(e, a) \mid G'_{\mathsf{tgt}}.\mathsf{mo}(e, a)\} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G_{\mathsf{src}}.\mathsf{ew} \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{ew}(a, e)\}
\end{aligned}
$$

Now we check the consistency of G'_{src}.

We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

Assume there is a $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle. We know there is no $G_{\mathsf{src}}.\mathsf{hb}; G_{\mathsf{src}}.\mathsf{eco}^?$ cycle, Hence the cycle involves event e. However, if event e introduces a $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$, then from the definition, there is a $G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^?$ cycle which is a contradiction. Hence G'_{src} satisfies (COH′) and G'_{src} is consistent.

Source Execution Construction. Next, we construct an execution $\mathsf{X}_t \in \mathsf{ex}_{\mathrm{WEAKESTMO}}(G_{\mathsf{tgt}})$.

If $\mathsf{X}_t.\mathsf{E}$ does not contain any event in C then we find the corresponding execution X_s such that $\mathsf{X}_s \in \mathsf{ex}_{\mathrm{WEAKESTMO}}(G_{\mathsf{src}})$ and $\mathsf{X}_s.\mathsf{E}$ contains no corresponding $\mathsf{St}_{\mathrm{NA}}(x, v)$ and $\mathsf{Ld}_{\mathrm{NA}}(x, v)$ events.

Else if an event $c \in C$ is in X_t, then we know that we can find an execution with $r, w \in \mathsf{X}_s.\mathsf{E}$ where $r.\mathsf{lab} = \mathsf{Ld}_{\mathrm{NA}}(x, \_)$ and $w.\mathsf{lab} = \mathsf{St}_{\mathrm{NA}}(x, \_)$. Thus X_s is as follows.

$$\mathsf{X}_s.\mathsf{E} = \mathsf{X}_t \uplus \uplus\{r, w \mid \mathsf{X}_t.\mathsf{E} \cap C \neq \emptyset\}$$

$$\mathsf{X}_s.\mathsf{po} = (\mathsf{X}_t.\mathsf{po}$$
$$\uplus\{(c,r),(r,w),(w,d) \mid (c,d) \in \mathsf{imm}(\mathsf{X}_t.\mathsf{po}) \cap (C \times D) \wedge r, w \in (G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E})\})^{+}$$

$$\mathsf{X}_s.\mathsf{rf} = \mathsf{X}_t.\mathsf{rf}\{(s,r) \mid r \in (G_{\mathsf{src}}.\mathsf{E} \setminus G_{\mathsf{tgt}}.\mathsf{E}) \cap \mathsf{codom}([C];\mathsf{imm}(G_{\mathsf{src}}.\mathsf{po})) \wedge G_{\mathsf{src}}.\mathsf{rf}(s,r)\}$$

$$\mathsf{X}_s.\mathsf{mo} = \mathsf{X}_t.\mathsf{mo} \uplus \{(a,w) \mid (a,w) \in (G_{\mathsf{src}}.\mathsf{mo} \cup G_{\mathsf{src}}.\mathsf{mo}^{-1}) \cap (\mathsf{X}_t.\mathsf{E} \times W)\}$$

Now we check the consistency of X_s.

We already know that X_t is consistent. We also know either $\mathsf{X}_s = \mathsf{X}_t$ or X_s has newly introduced r, w events. In that case, following the definition of X_s, the (Well-formed), (total-MO), (Coherence), (Atomicity) constraints also hold for X_s and hence X_s is consistent.

Same Behavior.

Now we check whether Behavior(X_s) = Behavior(X_t) holds. We consider the case where w is in X_s.

- In this case either s or s' is in X_s where $G_{\mathsf{src}}.\mathsf{ew}(s,s')$. In this case let $s.\mathsf{wval} = s'.\mathsf{wval} = v$. If s or s' is $\mathsf{X}_t.\mathsf{mo}$ maximal on x then $(x,v) \in$ Behavior(X_t). In this case is w is $\mathsf{X}_s.\mathsf{mo}$ maximal on x and hence $(x,v) \in$ Behavior(X_s).
- If s or s' is not $\mathsf{X}_t.\mathsf{mo}$ maximal then there exists w' such that $w'.\mathsf{wval} = v'$ and $(x,v') \in$ Behavior(X_t). In this case $\mathsf{X}_s.\mathsf{mo}(w,w')$ holds and and w' is $\mathsf{X}_s.\mathsf{mo}$ maximal. As a result, $(x,v') \in$ Behavior(X_s).

As a result, Behavior$|_x$ (X_s) = Behavior$|_x$ (X_t) holds in both cases. For locations $y \neq x$, Behavior$|_y$ (X_s) = Behavior$|_y$ (X_t) holds. As a result, Behavior(X_s) = Behavior(X_t) holds.

Race Preservation. Moreover, if X_t is racy, then the new write d does not introduce any $\mathsf{X}_s.\mathsf{sw}_{\mathsf{C11}}$ edge in X_s. Hence X_s is also racy. As a result, if the target execution has undefined behavior due to a data race, so does the source execution. □

G.5 Non-Adjacent Access Elimination (NA-OW)

Definition 11. A trace τ satisfies the *intermediate condition* for a location, x, which is written as GoodInterm$_x(\tau)$, if:

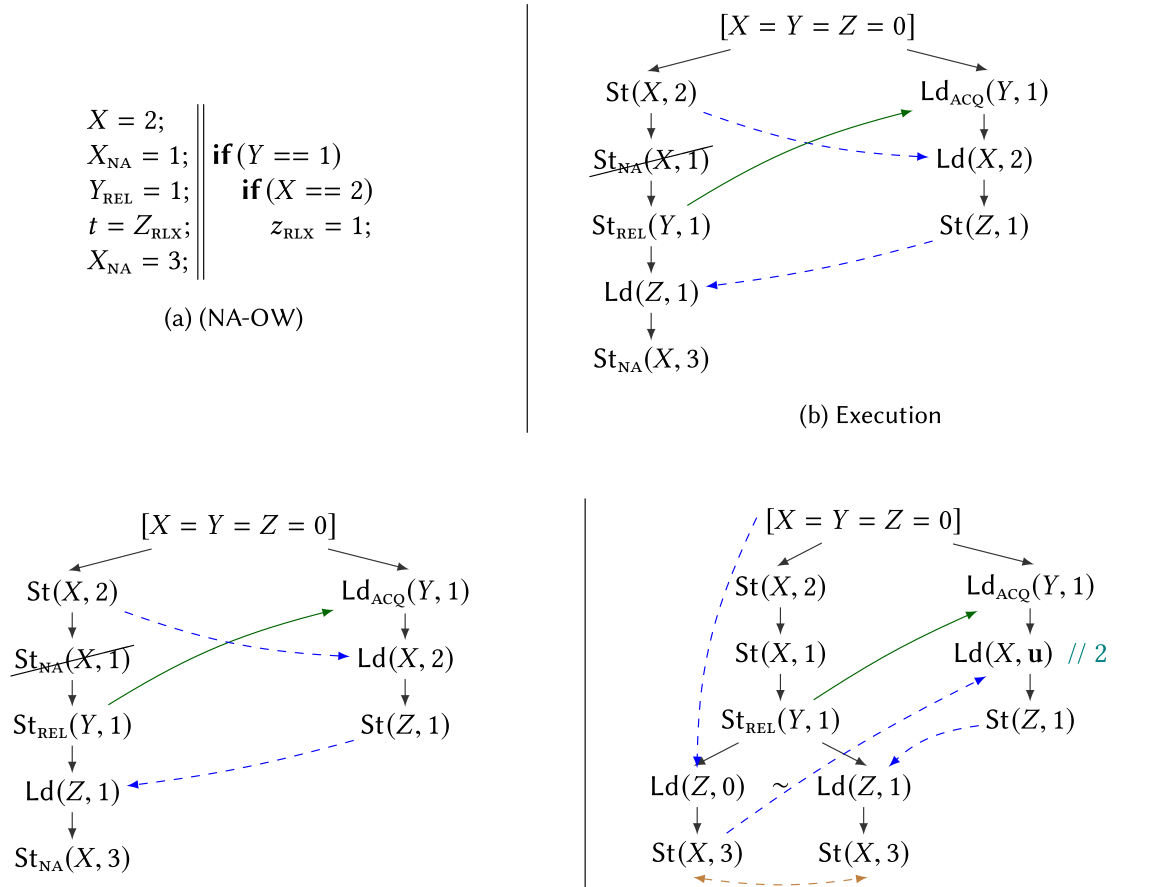
- it contains *no x-accesses*, i.e., $\tau \neq \tau_1 \cdot \mathcal{RW}_x \cdot \tau_2$ for all τ_1 and τ_2; and
- it contains *no rel-acq pairs*, i.e., $\tau \neq \tau_1 \cdot [\mathsf{Rel}] \cdot \tau_2 \cdot [\mathsf{Acq}] \cdot \tau_3$ for all traces τ_1, τ_2, and τ_3.

Let $\mathcal{E}_\tau$ be the events corresponding to τ. If $\mathcal{E}_\tau$ has no release access then $\mathsf{St}_{\mathsf{NA}}(x,v')$ could reorder with $\mathcal{E}_\tau$ and placed in adjacence with $\mathsf{St}_{\mathsf{NA}}(x,v)$. Then $\mathsf{St}_{\mathsf{NA}}(x,v')$ could be deleted by overwritten write (OW) transformation. But if $\mathcal{E}_\tau$ contains a release operation then $\mathsf{St}_{\mathsf{NA}}(x,v')$ cannot be reordered with $\mathcal{E}_\tau$. Hence in this proof we consider the cases where C contains release access. Before going to the proof we discuss a special case for WEAKESTMO-LLVM model.

Special Case. Given the program in consider the transformation deletes the $X_{\mathsf{NA}} = 1$ access and hence results in an taget execution as shown in . This execution has a defined behavior according to the WEAKESTMO-LLVM model as there is no write-write race in this execution.

The execution can be extracted from the target event structure in Fig. 35c.

Given this target event structure we cannot contruct the source event structure as once we introduce $\mathsf{St}_{\mathsf{NA}}(X,1)$, we cannot create $\mathsf{Ld}(X,2)$ directly.



(a) (NA-OW)

(b) Execution

(c) WEAKESTMO-LLVM target event structure

(d) WEAKESTMO-LLVM source event structure

Fig. 35. NA-OW example executions and WEAKESTMO-LLVM event structures.

However, note that, $\mathsf{Ld}(X, 2)$ is in read-write race with $\mathsf{St}_{\mathrm{NA}}(X, 3)$. Hence the program has undefined behavior in WEAKESTMO-C11 and in WEAKESTMO-LLVM the respective event may return $\mathbf{u}$ which can be evaluated to 2.

However, if $\mathsf{St}_{\mathrm{NA}}(X, 3)$ is appended after $\mathsf{Ld}(X, 2)$, then we cannot create $\mathsf{Ld}(X, \mathbf{u})$ in the source event structure directly. Hence G_{src} requires to create a $\mathsf{St}_{\mathrm{NA}}(X, _)$before $\mathsf{Ld}(X, \mathbf{u})$ as shown in .

Proof. Let W be the set of stores of thread i of G_{tgt} with label $\mathsf{St}_o(x, v)$, and whose po-prefix has some sequence of labels τ such that $\mathsf{St}_{\mathrm{NA}}(x, v')\cdot\tau\cdot\mathsf{St}_{\mathrm{NA}}(x, v) \notin \mathbb{P}_{\mathrm{src}}(i)$. Then, because of the relationship between the two programs, we know that for each such $w \in W$, $\mathsf{St}_{\mathrm{NA}}(x, v')\cdot\tau\cdot\mathsf{St}_{\mathrm{NA}}(x, v) \in \mathbb{P}_{\mathrm{src}}(i)$ for the appropriate τ.

Let
C be the set of first event in the sequence τ.
B be the set of immediate $G_{\mathrm{tgt}}.\mathsf{po}$-predecessor of C.
$F = G_{\mathrm{tgt}}.\mathsf{Rel}_{\neq x}$ are the release operations in τ.
W be the set of the respective $\mathsf{St}_o(x, v)$ labelled events and $W \subseteq \mathrm{codom}([F]; G_{\mathrm{tgt}}.\mathsf{po})$.
R be the set of reads such that $R \subseteq (\mathrm{codom}([B]; G_{\mathrm{tgt}}.\mathsf{po}; [F]G_{\mathrm{tgt}}.\mathsf{swe}; G_{\mathrm{tgt}}.\mathsf{hb}) \cap G_{\mathrm{tgt}}.\mathcal{R}_x)$ and $M : R \mapsto G_{\mathrm{src}}.\mathsf{E}$ maps a read in R to the corresponding read in source event structure. Let P be the

τ_x be the sub-sequence from $f \in F$ to $w \in W$ such that $G_{\mathsf{tgt}}.\mathsf{po}(f, w)$ holds and there is no $f' \in F$ such that $G_{\mathsf{tgt}}.\mathsf{po}(f', f)$.

$\mathsf{pc}(\tau_x)$ be the $G_{\mathsf{src}}.\mathsf{po}$-maximal event appended to the source event structure.

$EW(\tau_x)$ be the set of writes on x with label $\mathsf{St}_{\mathrm{NA}}(x, v)$ in G_{src}. The writes in $EW(\tau_x)$ are equal writes, that is, $\forall w_1, w_2 \in EW(\tau_x).G_{\mathsf{src}}.\mathsf{ew}(w_1, w_2)$ holds.

D be the set of events deleted from source event structure.

S be the events of τ_x that is, $S \subseteq \mathsf{codom}([F].G_{\mathsf{tgt}}.\mathsf{po}) \cup \mathsf{dom}(G_{\mathsf{tgt}}.\mathsf{po}; [W])$.

Source Event Structure Construction. To construct G_{src}, we follow the construction steps of G_{tgt}. For each target construction step that adds event e to G_{tgt} to get G'_{tgt}, we perform one or more corresponding steps going from G_{src} to G'_{src}. We do a case analysis on the event e of the target event structure.

Case $e \in C$:

We append a $\mathsf{St}_{\mathrm{NA}}(x, v')$ event d followed by event e as follows. The immediate $G_{\mathsf{tgt}}.\mathsf{po}$ predecessor of e is b.

Let s be the maximal-visible write on x w.r.t b, that is, $\mathsf{existsW}(G_{\mathsf{src}}, s, b)$ hold. We refer to the event s to create the mo relations to/from d.

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{d, e\} \quad \text{where } d.\mathsf{lab} = \mathsf{St}_{\mathrm{NA}}(x, v') \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(d, e)\} \uplus \{(b, d) \mid (b, e) \in G'_{\mathsf{tgt}}.\mathsf{po}\})^+ \\
G'_{\mathsf{src}}.\mathsf{jf} &= G'_{\mathsf{tgt}}.\mathsf{jf} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G_{\mathsf{src}}.\mathsf{mo} \uplus \{(s, d)\} \uplus \{(p, d) \mid G_{\mathsf{src}}.\mathsf{mo}(p, s)\} \uplus \{(d, p) \mid G_{\mathsf{src}}.\mathsf{mo}(s, p)\} \\
&\qquad \text{where } \mathsf{existsW}(G_{\mathsf{src}}, s, b). \\
G'_{\mathsf{src}}.\mathsf{ew} &= G_{\mathsf{src}}.\mathsf{ew} \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{ew}(a, e)\}
\end{aligned}
$$

Also we update D to $D \uplus \{d\}$. Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

From the definition, there is no $G_{\mathsf{src}}.\mathsf{hb}; G_{\mathsf{src}}.\mathsf{eco}^?$ as well as $G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^?$ cycle. Compared to G_{src} and G'_{tgt}, the additional $G'_{\mathsf{src}}.\mathsf{mo}$ edges are from and to the event d. Assume the mo edges to or from d creates a $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle. However, for each $G'_{\mathsf{src}}.\mathsf{mo}(d, a)$ or $G'_{\mathsf{src}}.\mathsf{mo}(a, d)$ already there exists $G'_{\mathsf{src}}.\mathsf{mo}(s, a)$ or $G'_{\mathsf{src}}.\mathsf{mo}(a, s)$ respectively. Thus event d as well as e results no new $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^?$ cycle and hence G'_{src} satisfies (COH′).

Case $e \in S$: Let e is in sequence τ_x. Two possibilities:

Subcase There exists an event e_s such that $\mathsf{imm}(G_{\mathsf{src}}.\mathsf{po})(\mathsf{pc}(\tau_x), e_s)$**:** $\mathsf{pc}' = \mathsf{pc}[\tau_x \mapsto e_s]$. In this case $G'_{\mathsf{src}} = G_{\mathsf{src}}$ and hence G'_{src} is consistent.

Subcase Otherwise: We take two steps where we first create an intermediate event structure G'' by appending e. Next, we append a sequence of events Q where a read r_c reads from a maximal visible write w_v in G_{src}, that is, $\mathsf{existsW}(G_{\mathsf{src}}, w_v, r_c)$ until we append an event $w_c = \mathsf{St}_{\mathrm{NA}}(x, v)$. Moreover, $\mathsf{pc}' = \mathsf{pc}[\tau_x \mapsto e]$.

Next, we append a sequence of events Q where a read r_c reads from a maximal visible write w_v in G_{src}, that is, $\mathsf{existsW}(G_{\mathsf{src}}, w_v, r_c)$ until we append an event $w_c = \mathsf{St}_{\mathrm{NA}}(x, v)$.

Thus G'_{src} is as follows:

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{e\} \cup \{Q\} \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(a,e) \mid G'_{\mathsf{tgt}}.\mathsf{po}(a,e)\} \\
&\quad \uplus \{(p,q) \mid (p = e \lor p \in Q) \land q \in Q \land p \neq Q\})^{+} \\
&\quad\ \text{for all } q \in Q \\
G'_{\mathsf{src}}.\mathsf{jf} &= G'_{\mathsf{tgt}}.\mathsf{jf} \uplus \{(a,e) \mid G'_{\mathsf{tgt}}.\mathsf{jf}(a,e)\} \\
&\quad \uplus \{(w_v, r_c) \mid r_c \in Q \land r_c \in \mathcal{R} \land \mathsf{existsW}(G_{\mathsf{src}}, w_v, r_c)\} \\
&\quad\ \text{for all } r_c \in Q \\
G'_{\mathsf{src}}.\mathsf{mo} &= G_{\mathsf{src}}.\mathsf{mo} \uplus \{(a,e) \mid G'_{\mathsf{tgt}}.\mathsf{mo}(a,e)\} \\
&\quad \uplus \{(a,q) \mid q \in \mathcal{W} \land a.\mathsf{loc} = q.\mathsf{loc} \land \neg G_{\mathsf{src}}.\mathsf{cf}(a,q) \\
&\quad \land (a \in G_{\mathsf{src}}.\mathsf{E} \lor G'_{\mathsf{src}}.\mathsf{po}(a,q))\} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G_{\mathsf{src}}.\mathsf{ew} \uplus \{(a,e) \mid G'_{\mathsf{tgt}}.\mathsf{ew}(a,e)\} \uplus \{(w', w_c) \mid w' \in EW(\tau_x)\}
\end{aligned}
$$

and finally we update $EW(\tau_x)$, that is, $EW(\tau_x) = EW(\tau_x) \uplus \{w_c\}$.

Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

From the definition, there is no $G_{\mathsf{src}}.\mathsf{hb}; G_{\mathsf{src}}.\mathsf{eco}^{?}$ as well as $G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^{?}$ cycle. Compared to G_{src} and G'_{tgt}, the additional $G'_{\mathsf{src}}.\mathsf{hb}$ and $G_{\mathsf{src}}.\mathsf{eco}$ edges are from and to the event $\{e\} \cup Q$. The edge from/to e does not create new $G'_{\mathsf{src}}.\mathsf{hb}; G'_{\mathsf{src}}.\mathsf{eco}^{?}$ cycle as there is no $G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^{?}$ cycle. Also the outgoing $G'_{\mathsf{src}}.\mathsf{hb}$ and $G_{\mathsf{src}}.\mathsf{eco}$ edges from events in Q are only to other events in Q. In cosequence, there is no $G'_{\mathsf{tgt}}.\mathsf{hb}; G'_{\mathsf{tgt}}.\mathsf{eco}^{?}$ cycle to/from Q events. Thus G'_{src} satisfies (COH′) and G'_{src} is consistent.

Case $e \in R$:

In this case event e reads from a visible write w_1 which is now overwritten. w_1 has a G'_{tgt}.po-successor sequence τ which includes $f \in F$ suh that $G'_{\mathsf{tgt}}.\mathsf{po}(w_1, f)$. From the construction, f has a G_{src}.po event w_c such that $w_c.\mathsf{lab} = \mathsf{St}_{\textsc{na}}(x, v)$. Consider we append event r in source event structure corresponding to e.

Following the WEAKESTMO-C11 model, if we append an event corresponding to e it results in race and hence the source has undefined behavior. Hence the transformation is correct.

Now we consider the WEAKESTMO-LLVM model. If $r \in \mathsf{U}$, then there is a write-write race and in that case the source program has undefined behavior. Hence the transformation is correct.

The according to WEAKESTMO-LLVM read-write race has define behavior. Hence we continue the event structure construction when r is a load, that is, $r \in \mathsf{Ld}$.

We append r to the G_{src} as follows:

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{r\} \quad \text{where } r.\mathsf{lab} = \mathsf{Ld}(x, \mathbf{u})\text{which we evaluate } \mathbf{u} \text{ to } w_1.\mathsf{wval}. \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(a,r) \mid G'_{\mathsf{tgt}}.\mathsf{po}(a,e)\})^{+} \\
G'_{\mathsf{src}}.\mathsf{jf} &= G'_{\mathsf{tgt}}.\mathsf{jf} \uplus \{(w_c, r)\} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G_{\mathsf{src}}.\mathsf{mo} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G_{\mathsf{src}}.\mathsf{ew}
\end{aligned}
$$

Also we update the mapping $M' = M[e \mapsto r]$.

Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

From the definition, there is no $G_{\mathrm{src}}.\mathsf{hb}; G_{\mathrm{src}}.\mathsf{eco}^?$ cycle. So any new $G'_{\mathrm{src}}.\mathsf{hb}; G'_{\mathrm{src}}.\mathsf{eco}^?$ cycle involves r. The incoming edges to r is $G'_{\mathrm{src}}.\mathsf{po}$, $G'_{\mathrm{src}}(w_c, r)$ and the outgoing edges are $G'_{\mathrm{src}}.\mathsf{fr}$ edges when $w_c \in G'_{\mathrm{tgt}}.\mathsf{E}$as well. These edges cannot contitute a $G'_{\mathrm{src}}.\mathsf{hb}; G'_{\mathrm{src}}.\mathsf{eco}^?$ cycle as there is no $G'_{\mathrm{tgt}}.\mathsf{hb}; G'_{\mathrm{tgt}}.\mathsf{eco}^?$ cycle involving w_c. As a result, G'_{src} preserves (COH′) and G'_{src} is consistent.

Case $e \in W$:

Either there already exists a write event $w_c \in EW(\tau_x)$ with $w_c.\mathsf{lab} = \mathsf{St}_{\mathrm{NA}}(x, v)$ such that $\mathsf{imm}(G_{\mathrm{src}}.\mathsf{po})(\mathsf{pc}(\tau_x), w_c)$ or we append event e.

Subcase $\exists w_c \in EW(\tau_x)$ such that $w_c.\mathsf{lab} = \mathsf{St}_{\mathbf{NA}}(x, v)$, $\mathsf{imm}(G_{\mathrm{src}}.\mathsf{po})(\mathsf{pc}(\tau_x), w_c)$:
In this case $\mathsf{pc}' = \mathsf{pc}[\tau_x \mapsto w_c]$ and G'_{src} is as follows:

$$
\begin{aligned}
G'_{\mathrm{src}}.\mathsf{E} &= G_{\mathrm{src}}.\mathsf{E}\\
G'_{\mathrm{src}}.\mathsf{po} &= G_{\mathrm{src}}.\mathsf{po}\\
G'_{\mathrm{src}}.\mathsf{jf} &= G_{\mathrm{src}}.\mathsf{jf}\\
G'_{\mathrm{src}}.\mathsf{mo} &= G_{\mathrm{src}}.\mathsf{mo}\\
G'_{\mathrm{src}}.\mathsf{ew} &= G_{\mathrm{src}}.\mathsf{ew} \uplus \{(a, w_c) \mid G'_{\mathrm{tgt}}.\mathsf{ew}(a, e)\}
\end{aligned}
$$

Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

From the definition, there is no $G_{\mathrm{src}}.\mathsf{hb}; G_{\mathrm{src}}.\mathsf{eco}^?$ cycle. So any new $G'_{\mathrm{src}}.\mathsf{hb}; G'_{\mathrm{src}}.\mathsf{eco}^?$ cycle involves new outgoing $G'_{\mathrm{src}}.\mathsf{rf}$ from w_c. However, G'_{tgt} also has corresponding outgoing $G'_{\mathrm{tgt}}.\mathsf{rf}$ edge from e and there is no $G'_{\mathrm{tgt}}.\mathsf{hb}; G'_{\mathrm{tgt}}.\mathsf{eco}^?$ cycle involving e. Hence there is no $G'_{\mathrm{src}}.\mathsf{hb}; G'_{\mathrm{src}}.\mathsf{eco}^?$ cycle involving w_c. As a result, G'_{src} satisfies (COH′) and G'_{src} is consistent.

Subcase Otherwise: We append e to G_{src} and construct G'_{src} as follows where $\mathsf{pc}'(\tau_x) = e$.

$$
\begin{aligned}
G'_{\mathrm{src}}.\mathsf{E} &= G_{\mathrm{src}}.\mathsf{E} \uplus \{e\}\\
G'_{\mathrm{src}}.\mathsf{po} &= (G_{\mathrm{src}}.\mathsf{po} \uplus \{(\mathsf{pc}(\tau_x), e)\})^+\\
G'_{\mathrm{src}}.\mathsf{jf} &= G'_{\mathrm{tgt}}.\mathsf{jf}\\
G'_{\mathrm{src}}.\mathsf{mo} &= G_{\mathrm{src}}.\mathsf{mo} \uplus \{(a, e) \mid G'_{\mathrm{tgt}}.\mathsf{mo}(a, e)\} \uplus \{(e, a) \mid G'_{\mathrm{tgt}}.\mathsf{po}(e, a)\}\\
&\quad \uplus \{(w, e) \mid w.\mathsf{lab} = \mathsf{St}_{\mathrm{NA}}(x, v') \wedge w \in \mathsf{codom}([B]; G_{\mathrm{src}}.\mathsf{po})\\
&\quad \cap \mathsf{dom}(G_{\mathrm{src}}.\mathsf{po}; [C]) \wedge G_{\mathrm{src}}.\mathsf{po}(w, \mathsf{pc}(\tau_x))\}\\
G'_{\mathrm{src}}.\mathsf{ew} &= G_{\mathrm{src}}.\mathsf{ew} \uplus \{(a, e) \mid G'_{\mathrm{tgt}}.\mathsf{ew}(a, e)\}
\end{aligned}
$$

Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

From the definition, there is no $G_{\mathrm{src}}.\mathsf{hb}; G_{\mathrm{src}}.\mathsf{eco}^?$ cycle. So any new $G'_{\mathrm{src}}.\mathsf{hb}; G'_{\mathrm{src}}.\mathsf{eco}^?$ cycle involves event e. However, if there is any outgoing $G'_{\mathrm{src}}.\mathsf{mo}$ edge from e then there is a write-write race and hence the source program has undefined behavior. Hence there is no $G'_{\mathrm{src}}.\mathsf{hb}; G'_{\mathrm{src}}.\mathsf{eco}^?$ cycle involving e. As a result, G'_{src} satisfies (COH′) and G'_{src} is consistent.

Case $e \in G'_{\mathrm{tgt}}.\mathsf{E} \setminus (C \cup S \cup R \cup W)$:

We construct the G'_{src} as follows:

$$
\begin{aligned}
G'_{\text{src}}.\mathsf{E} &= G_{\text{src}}.\mathsf{E} \uplus \{e\}\\
G'_{\text{src}}.\mathsf{po} &= (G_{\text{src}}.\mathsf{po} \uplus \{(a,e) \mid G'_{\text{tgt}}.\mathsf{po}(a,e)\})^{+}\\
G'_{\text{src}}.\mathsf{jf} &= G'_{\text{tgt}}.\mathsf{jf} \uplus \{(a,e) \mid G'_{\text{tgt}}.\mathsf{jf}(a,e)\}\\
G'_{\text{src}}.\mathsf{mo} &= G_{\text{src}}.\mathsf{mo} \uplus \{(a,e) \mid G'_{\text{tgt}}.\mathsf{mo}(a,e)\}\\
&\quad \uplus \{(d,e) \mid d \in D \land G_{\text{tgt}}.\mathsf{mo}(s,e) \land \mathsf{existsW}(G'_{\text{src}}, s, d)\}\\
&\quad \uplus \{(e,d) \mid d \in D \land G_{\text{tgt}}.\mathsf{mo}(e,s) \land \mathsf{existsW}(G'_{\text{src}}, s, d)\}\\
&\quad \uplus \{(e,c) \mid c \in G'_{\text{src}}.\mathsf{E} \setminus G'_{\text{tgt}}.\mathsf{E} \land c.\mathsf{loc} = e.\mathsf{loc} \land \neg G'_{\text{src}}.\mathsf{cf}(e,e)\}\\
G'_{\text{src}}.\mathsf{ew} &= G_{\text{src}}.\mathsf{ew} \uplus \{(a,e) \mid G'_{\text{tgt}}.\mathsf{ew}(a,e)\}
\end{aligned}
$$

Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ) constraints immediately hold. It remains to show that G'_{src} satisfies (COH′).

From the definition, there is no $G_{\text{src}}.\mathsf{hb}; G_{\text{src}}.\mathsf{eco}^{?}$ cycle. So any new $G'_{\text{src}}.\mathsf{hb}; G'_{\text{src}}.\mathsf{eco}^{?}$ cycle involves event $d \in D$ or the events in $G'_{\text{src}}.\mathsf{E} \setminus G'_{\text{tgt}}.\mathsf{E}$. However, following the definition, if there is any new $G'_{\text{src}}.\mathsf{hb}; G'_{\text{src}}.\mathsf{eco}^{?}$ cycle involving event d then there is a cycle involving write event s where $\mathsf{existsW}(G'_{\text{src}}, s, d)$. In that case there is also $G'_{\text{tgt}}.\mathsf{hb}; G'_{\text{tgt}}.\mathsf{eco}^{?}$ cycle which is a contradiction. The writes in $G'_{\text{src}}.\mathsf{E} \setminus G'_{\text{tgt}}.\mathsf{E}$ have no outgoing $G'_{\text{src}}.\mathsf{mo} \setminus G'_{\text{src}}.\mathsf{po}$ edge and hence cannot create a $G'_{\text{src}}.\mathsf{hb}; G'_{\text{src}}.\mathsf{eco}^{?}$ cycle. The reads in $G'_{\text{src}}.\mathsf{E} \setminus G'_{\text{tgt}}.\mathsf{E}$ may have outgoing $G'_{\text{src}}.\mathsf{fr}$ edges. However, if any such $G'_{\text{src}}.\mathsf{fr}$ edge creates a cycle then following the definition, there is already a $G_{\text{src}}.\mathsf{hb}; G_{\text{src}}.\mathsf{eco}^{?}$ cycle which is a contradiction. Hence G'_{src} satisfies (COH′) and G'_{src} is consistent.

Source Execution Construction. Next, we construct an execution $\mathsf{X}_t \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\text{tgt}})$.

If $W \subseteq (G_{\text{tgt}}.\mathsf{E} \setminus \mathsf{X}_t.\mathsf{E})$, then we find the corresponding execution $\mathsf{X}_s \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\text{src}})$ such that X_s contains no event created for $\mathsf{store}_{o'}(x, v')$. Else if an event $w \in W$ is in X_t, then we know that we can find an execution with $w \in \mathsf{X}_s.\mathsf{E}$ and $\mathsf{X}_s.\mathsf{E}$ also contains an event $d \in D$ where $d.\mathsf{lab} = \mathsf{St}_{\text{NA}}(x, v')$. Also let $r \in R \cap \mathsf{X}_t.\mathsf{E}$. Thus X_s is as follows.

$$
\begin{aligned}
\mathsf{X}_s.\mathsf{E} &= \mathsf{X}_t.\mathsf{E} \uplus \{d \mid \mathsf{X}_t.\mathsf{E} \cap W \neq \emptyset\} \setminus \{r \mid r \in R \cap \mathsf{X}_t.\mathsf{E}\} \uplus \{M(r) \mid r \in R \cap \mathsf{X}_t.\mathsf{E}\}\\
\mathsf{X}_s.\mathsf{po} &= (\mathsf{X}_t.\mathsf{po} \uplus \{(b,d),(d,c) \mid (b,c) \in \mathsf{imm}(\mathsf{X}_t.\mathsf{po}) \cap (B \times C) \land d \in (G_{\text{src}}.\mathsf{E} \setminus G_{\text{tgt}}.\mathsf{E})\}\\
&\quad \setminus \{(p,r) \mid \mathsf{X}_t.\mathsf{po}(p,r) \land p \notin R \land r \in R \cap \mathsf{X}_t.\mathsf{E}\}\\
&\quad \uplus \{(p, M(r)) \mid \mathsf{X}_t.\mathsf{po}(p,r) \land p \notin R \land r \in R \cap \mathsf{X}_t.\mathsf{E}\})^{+}\\
\mathsf{X}_s.\mathsf{rf} &= \mathsf{X}_t.\mathsf{rf} \setminus \{(a,r) \mid r \in R\} \uplus \{(w, M(r)) \mid G_{\text{src}}.\mathsf{rf}(w, M(r)) \land r \in R \land w \in \mathsf{X}_s.\mathsf{E}\}\\
\mathsf{X}_s.\mathsf{mo} &= \mathsf{X}_t.\mathsf{mo} \uplus \{(d,w) \mid d \in D \land w \in \mathsf{codom}([D]; G_{\text{src}}.\mathsf{mo}) \cap \mathsf{X}_s.\mathsf{E}\}\\
&\quad \uplus \{(w,d) \mid d \in D \land w \in \mathsf{dom}(G_{\text{src}}.\mathsf{mo}; [D]) \cap \mathsf{X}_s.\mathsf{E}\}
\end{aligned}
$$

Source Execution Consistency. Now we check the consistency of X_s.

- Following the definition of X_s the (Well-formed) is satisfied.
- We know that X_t follows (total-MO). The additional write d introduced in X_s has the label $\mathsf{St}_{\text{NA}}(x, v')$. However, from the definition of G_{src} and X_s, event d preserves (total-MO).
- Assume (Atomicity) does not hold in X_s. We know that (Atomicity) holds in X_t. Hence (Atomicity) is violated due to event d. In that case there exists $u \in \mathsf{X}_s.\mathsf{U}_x$ such that $\mathsf{X}_s.\mathsf{fr}(u, d)$

and $X_s.\mathsf{mo}(d, u)$. However, in this case there is a write-write race and hence the source program has undefined behavior which is a contradiction. Hence (Atomicity) holds in X_s.

- Now we check if (SC) holds. As $d \notin \mathsf{SC}$, it introduces no new $[\mathsf{SC}]; X_s.\mathsf{hb}_{\mathsf{C11}}; [\mathsf{SC}]$ path compared to X_t. We also know that SC holds on X_t. As a result, X_s also preserves SC.

Thus X_s is consistent and $X \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{src}})$ holds.

Same Behavior.

For locations $y \neq x$, we have $X_s.E_y = X.E_y$ and so $\mathsf{Behavior}(X_s)|_y = \mathsf{Behavior}(X_t)|_y$ trivially holds. Now we check whether $\mathsf{Behavior}(X_s)|_x = \mathsf{Behavior}(X_t)|_x$ holds. Note that any newly introduced event $d \in X_s.E \setminus X_t.E$ is not $X_s.\mathsf{mo}$ maximal, because in that case there exists a store $\mathsf{St}_{\text{NA}}(x, v)$ which is $X_s.\mathsf{mo}$ after d. Hence $\mathsf{Behavior}(X_s) = \mathsf{Behavior}(X_t)$ holds.

Race Preservation. Moreover, if X_t is racy, then the new write d does not introduce any $X_s.\mathsf{sw}_{\mathsf{C11}}$ edge in X_s. Hence X_s is also racy. As a result, if the target execution has undefined behavior due to a data race, so does the source execution.

□

H PROOF OF CORRECTNESS OF SPECULATIVE LOAD

Theorem 8. *The transformation $\epsilon \rightsquigarrow \mathsf{Ld}_o(x, \_)$ is correct in* WEAKESTMO-LLVM.

Proof. Let $R \subset G_{\mathsf{tgt}}.\mathsf{E}$ be the set of introduced events with label $\mathsf{Ld}_o(x, v)$ in the target event structure G_{tgt} such that

Let R be the set of events of thread i of G_{tgt} with label $\mathsf{Ld}_o(x, v)$ such that $\tau \cdot \mathsf{Ld}_o(x, v) \cdot \tau' \notin \mathbb{P}_{\mathsf{src}}(i)$. Then, because of the relationship between the two programs, we know that for each such $r \in R$, $\tau \cdot \tau' \in \mathbb{P}_{\mathsf{src}}(i)$ holds. Let C be the immediate $G_{\mathsf{tgt}}.\mathsf{po}$ successors of R events.

Source Event Structure Construction.

To construct G_{src}, we follow the construction steps of G_{tgt}. For each target construction step that adds event e to G_{tgt} to get G'_{tgt}, we perform one or more corresponding steps going from G_{src} to G'_{src}. We do a case analysis on the event e of the target event structure.

Case $e \in R$:

In this case $G'_{\mathsf{src}} = G_{\mathsf{src}}$ and G'_{src} is consistent as G_{src} is consistent.

Case $e \in C$: In this case we append e to the event in C as follows:

$$
\begin{aligned}
G'_{\mathsf{src}}.\mathsf{E} &= G_{\mathsf{src}}.\mathsf{E} \uplus \{e\} \\
G'_{\mathsf{src}}.\mathsf{po} &= (G_{\mathsf{src}}.\mathsf{po} \uplus \{(c, e) \mid (e, e) \in [C]; \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po}); [R]; \mathsf{imm}(G'_{\mathsf{tgt}}.\mathsf{po})\})^+ \\
G'_{\mathsf{src}}.\mathsf{jf} &= G_{\mathsf{src}}.\mathsf{jf} \uplus \{(a, e) \mid G'_{\mathsf{tgt}}.\mathsf{jf}(a, e)\} \\
G'_{\mathsf{src}}.\mathsf{mo} &= G'_{\mathsf{tgt}}.\mathsf{mo} \\
G'_{\mathsf{src}}.\mathsf{ew} &= G'_{\mathsf{tgt}}.\mathsf{ew}
\end{aligned}
$$

Now we check the consistency of G'_{src}. We already know that G_{src} and G'_{tgt} is consistent. Following the construction of G'_{src}, the (CF), (CFJ), (VISJ), (ICF), (ICFJ), (COH′) constraints immediately hold.

Case $e \in G'_{\mathsf{tgt}}.\mathsf{E} \setminus (C \cup R)$:

Source Execution Construction. Next, we construct an execution $\mathsf{X}_t \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{tgt}})$. If $R \subseteq (G_{\mathsf{tgt}} \setminus \mathsf{X}_t.\mathsf{E})$, then we find the corresponding execution $\mathsf{X}_s \in \mathsf{ex}_{\text{WEAKESTMO}}(G_{\mathsf{src}})$ such that X_s contains no event created for $\mathsf{Ld}_o(x, v)$. Else if an event $r \in R$ is in X_t, then we know that we can find an execution with $r \notin \mathsf{X}_s.\mathsf{E}$. Thus X_s is as follows.

$$
\begin{aligned}
\mathsf{X}_s.\mathsf{E} &= \mathsf{X}_t.\mathsf{E} \setminus R \\
\mathsf{X}_s.\mathsf{po} &= \mathsf{X}_t.\mathsf{po} \setminus \{(a, b) \mid a \in R \vee b \in R\} \\
\mathsf{X}_s.\mathsf{rf} &= \mathsf{X}_t.\mathsf{rf} \setminus \{(a, b) \mid a \in R \vee b \in R\} \\
\mathsf{X}_s.\mathsf{mo} &= \mathsf{X}_t.\mathsf{mo}
\end{aligned}
$$

Source Execution Consistency. Now we check the consistency of X_s.

Since X_t is consistent, the (Well-formed), (total-MO), (Coherence), (Atomicity), (SC) constraints also hold for X_s.

Same Behavior. The R events are loads and hence do not affect program behavior. Hence, $\mathsf{Behavior}(\mathsf{X}_s) = \mathsf{Behavior}(\mathsf{X}_t)$ holds.

Race Preservation. The R events may introduce new read-write races in the target execution compared to the source execution. This is not correct in WEAKESTMO-C11 model, but it is fine in the WEAKESTMO-LLVM model. ◻