

# Appendix of DimSum: A Decentralized Approach to Multi-language Semantics and Verification

MICHAEL SAMMLER, MPI-SWS, Germany

SIMON SPIES, MPI-SWS, Germany

YOUNGJU SONG, MPI-SWS, Germany

EMANUELE D'OSUALDO, MPI-SWS, Germany

ROBBERT KREBBERS, Radboud University Nijmegen, The Netherlands

DEEPAK GARG, MPI-SWS, Germany

DEREK DREYER, MPI-SWS, Germany

## A COMBINATORS

**Filter.** The filter combinator  $M \setminus_{\sigma} M' \triangleq (S_{\text{filter}} \times S_M \times S_{M'}, \rightarrow_{\text{filter}}, (\sigma, \sigma_M^0, \sigma_{M'}^0))$  takes in a module  $M \in \text{Module}(E_1)$  and a filter  $M' \in \text{Module}(\text{FilterEvents}(E_1, E_2))$  and then produces a module with events drawn from  $E_2$ . The states of the filter combinator are given by  $S_{\text{filter}} \triangleq \{P, F\} \cup \{P(e) \mid e \in E_1\} \cup \{F(e) \mid e \in E_1\}$  and the transitions are depicted in Fig. 1. The events of the filter module are drawn from the following set:

$$\begin{aligned} \text{FilterEvents}(E_1, E_2) \triangleq & \{ \text{FromInner}(e_1) \mid e_1 \in E_1 \} \cup \{ \text{ToInner}(e_1) \mid e_1 : \text{option}(E_1) \} \cup \\ & \{ \text{ToEnv}(e_2) \mid e_2 : E_2 \} \cup \{ \text{FromEnv}(e_2) \mid e_2 : E_2 \} \end{aligned}$$

The event  $\text{FromInner}(e_1)$  means that  $M'$  is willing to accept  $e_1$  from  $M$ . The event  $\text{ToInner}(e_1)$  means that  $M'$  wants to return control to the module  $M$ , optionally sending it the event  $e_1$ . Sending  $e_1$  to  $M$  means that  $M$  all visible transitions of the inner module  $M$  except ones emitting event  $e_1$  are blocked. The event  $\text{ToEnv}(e_2)$  means that  $M'$  wants to emit  $e_2$  to the environment, and  $\text{FromEnv}(e_2)$  means that  $M'$  is willing to accept  $e_2$  from the environment. Note that, while there is a difference between the intuition for  $\text{ToEnv}(e_2)$  and  $\text{FromEnv}(e_2)$ , both events are treated the same by  $M \setminus M'$  as DimSum does not distinguish between incoming and outgoing events.

**Linking.** The linking operator  $M_1 \oplus_X M_2$  is defined on modules  $M_1, M_2 \in \text{Module}(E_{?!})$  where  $E_{?!}$  is (an event type that is isomorphic to)  $E \times \{?, !\}$ . The parameter  $X = (S, \rightsquigarrow, s^0)$  determines how the events are linked. It consists of a set of linking-internal states  $S$ , an initial state  $s^0 \in S$ , and a relation  $\rightsquigarrow \subseteq (D \times S \times E) \times ((D \times S \times E) \cup \{\zeta\})$  describing how events should be translated. Formally, linking can be defined as  $M_1 \oplus_X M_2 \triangleq M_1 \times M_2 \setminus_{\rho} \text{link}_X$ .<sup>1</sup> The module  $\text{link}_X$  is defined as  $\text{link}_X \triangleq (S_{\text{link}} \times S_X, \rightarrow_{\text{link}}, (\text{Wait}, s_X^0))$  where  $S_{\text{link}} \triangleq (\{\text{Wait}, \text{Ub}\} \cup \{\text{ToEnv}(e, \sigma), \text{FromEnv}(e, \sigma) \mid e \in E_{?!}, \sigma \in S_{\text{link}}\}) \cup \{\text{ToInner}(e) \mid e \in \text{option}(E_{?!})\}$  and  $\rightarrow_{\text{link}}$  is defined in Fig. 2.

<sup>1</sup>The Coq development defines linking via more low-level combinators that we omit from the presentation here. Also the Coq development allows undefined behavior via a Boolean on the right side of  $\rightsquigarrow$  instead of a separate  $\zeta$  result.

---

Authors' addresses: Michael Sammler, MPI-SWS, Saarland Informatics Campus, Germany, msammler@mpi-sws.org; Simon Spies, MPI-SWS, Saarland Informatics Campus, Germany, spies@mpi-sws.org; Youngju Song, MPI-SWS, Saarland Informatics Campus, Germany, youngju@mpi-sws.org; Emanuele D'Ossualdo, MPI-SWS, Saarland Informatics Campus, Germany, dosualdo@mpi-sws.org; Robbert Krebbers, Radboud University Nijmegen, The Netherlands, mail@robbertkrebbers.nl; Deepak Garg, MPI-SWS, Saarland Informatics Campus, Germany, dg@mpi-sws.org; Derek Dreyer, MPI-SWS, Saarland Informatics Campus, Germany, dreyer@mpi-sws.org.

$$\begin{array}{c}
\text{FILTER-STEP-PROG-NONE} \\
\frac{\sigma = P \vee \sigma = P(e) \quad \sigma_1 \xrightarrow{\tau} \Sigma}{(\sigma, \sigma_1, \sigma_2) \xrightarrow{\tau}_{\text{filter}} \{(\sigma, \sigma'_1, \sigma_2) \mid \sigma'_1 \in \Sigma\}} \\
\\
\text{FILTER-STEP-PROG-RECV} \\
\frac{\sigma_1 \xrightarrow{e} \Sigma}{(P(e), \sigma_1, \sigma_2) \xrightarrow{\tau}_{\text{filter}} \{(P, \sigma'_1, \sigma_2) \mid \sigma'_1 \in \Sigma\}} \\
\\
\text{FILTER-STEP-FILTER-NONE} \\
\frac{\sigma = F \vee \sigma = F(e) \quad \sigma_2 \xrightarrow{\tau} \Sigma}{(\sigma, \sigma_1, \sigma_2) \xrightarrow{\tau}_{\text{filter}} \{(\sigma, \sigma_1, \sigma'_2) \mid \sigma'_2 \in \Sigma\}} \\
\\
\text{FILTER-STEP-PROG} \\
\frac{\sigma_1 \xrightarrow{e} \Sigma}{(P, \sigma_1, \sigma_2) \xrightarrow{\tau}_{\text{filter}} \{(F(e), \sigma'_1, \sigma_2) \mid \sigma'_1 \in \Sigma\}} \\
\\
\text{FILTER-STEP-FILTER-FROM-INNER} \\
\frac{\sigma_2 \xrightarrow{\text{FromInner}(e)} \Sigma}{(F(e), \sigma_1, \sigma_2) \xrightarrow{\tau}_{\text{filter}} \{(F, \sigma_1, \sigma'_2) \mid \sigma'_2 \in \Sigma\}} \\
\\
\text{FILTER-STEP-FILTER-TO-INNER} \\
\frac{\sigma_2 \xrightarrow{\text{ToInner}(e)} \Sigma}{(F, \sigma_1, \sigma_2) \xrightarrow{\tau}_{\text{filter}} \{(if\ e = \text{Some}(e')\ \text{then}\ P(e')\ \text{else}\ P, \sigma_1, \sigma'_2) \mid \sigma'_2 \in \Sigma\}} \\
\\
\text{FILTER-STEP-FILTER-TO-ENV} \\
\frac{\sigma_2 \xrightarrow{\text{ToEnv}(e)} \Sigma}{(F, \sigma_1, \sigma_2) \xrightarrow{e}_{\text{filter}} \{(F, \sigma_1, \sigma'_2) \mid \sigma'_2 \in \Sigma\}} \\
\\
\text{FILTER-STEP-FILTER-FROM-ENV} \\
\frac{\sigma_2 \xrightarrow{\text{FromEnv}(e)} \Sigma}{(F, \sigma_1, \sigma_2) \xrightarrow{e}_{\text{filter}} \{(F, \sigma_1, \sigma'_2) \mid \sigma'_2 \in \Sigma\}}
\end{array}$$

Fig. 1. Definition of  $\rightarrow_{\text{filter}}$ .

**(Kripke) wrappers.** The combinator  $[M]_X$  translates a module with events  $E_1$  to a module with events  $E_2$ . This combinator is parametrized by  $X = (S, \mathcal{R}, \leftarrow, \rightarrow, s^0, F^0)$  where  $S$  is a set of states and  $s^0$  is an initial state ( $\mathcal{R}$  is explained below). These states were omitted in the main paper for simplicity. They do not give additional expressive power but make writing the wrapper  $[\cdot]_{r \Leftrightarrow a}$  more pleasant. The relations  $\leftarrow$  and  $\rightarrow$  describe how the wrapper transforms the incoming and outgoing events. Concretely,  $\leftarrow$  describes how to translate an event  $e_2 \in E_2$  to an event  $e_1 \in E_1$  and  $\rightarrow$  describes the translation from  $e'_1 \in E_1$  to  $e'_2 \in E_2$ .

As mentioned in the paper, these relations are separation logic relations. Which separation logic the relations are defined in is determined by the parameter  $X$  of the wrapper. In the paper, it contains an arbitrary separation logic  $\mathcal{L}$  as one of its components. However, for our instantiations of the wrapper, we are only interested in instances of the separation logic Iris [Jung et al. 2015]. Thus, instead of an arbitrary separation logic  $\mathcal{L}$ , we parameterize the wrapper by a *resource algebra*  $\mathcal{R}$  and use the separation logic  $\mathcal{L} = \text{UPred}(\mathcal{R})$  where  $\text{UPred}(\mathcal{R})$  is Iris’s logic of uniform predicates [Jung et al. 2018]. The separation logic relations  $\leftarrow$  and  $\rightarrow$  are of type  $E_1 \times S \times E_2 \times S \rightarrow \text{UPred}(\mathcal{R})$ . The proposition  $F^0 : \text{UPred}(\mathcal{R})$  denotes the initial set of resources owned by the wrapper.

We define  $[M]_X \triangleq M \Vdash_{\mathbb{F}} [\text{wrap}(s^0, F^0)]_s$  where the filter module is given by the following Spec program:<sup>2</sup>

```

wrap( $s_2, F_2$ )  $\triangleq_{\text{coind}}$ 
   $\exists e_2; \text{vis}(\text{FromEnv}(e_2)); \forall e_1, s_1, F_1; \text{assume}(\text{sat}(F_1 * F_2 * (e_1, s_1) \leftarrow (e_2, s_2))); \text{vis}(\text{ToInner}(e_1));$ 
   $\exists e'_1; \text{vis}(\text{FromInner}(e'_1)); \exists e'_2, s'_2, F'_2; \text{assert}(\text{sat}(F_1 * F'_2 * (e'_1, s_1) \rightarrow (e'_2, s'_2))); \text{vis}(\text{ToEnv}(e'_2));$ 
  wrap( $s'_2, F'_2$ )

```

$$\text{to}(d, e) = \begin{cases} \text{ToInner}(\text{left}(e?, L)) & \text{if } d = L \\ \text{ToInner}(\text{right}(e?, R)) & \text{else if } d = R \\ \text{ToEnv}(e!, \text{ToInner}(\text{None})) & \text{else if } d = E \end{cases}$$

$$\begin{array}{c} \text{LINK-STEP-WAIT-L} \\ \frac{(L, s, e) \rightsquigarrow (d, s', e')}{(\text{Wait}, s) \xrightarrow{\text{FromInner}(\text{left}(e!, d))} \text{link} \{(\text{to}(d, e'), s')\}} \\ \\ \text{LINK-STEP-WAIT-R} \\ \frac{(R, s, e) \rightsquigarrow (d, s', e')}{(\text{Wait}, s) \xrightarrow{\text{FromInner}(\text{right}(e!, d))} \text{link} \{(\text{to}(d, e'), s')\}} \\ \\ \text{LINK-STEP-WAIT-N} \\ \frac{(E, s, e) \rightsquigarrow (d, s', e')}{(\text{Wait}, s) \xrightarrow{\text{FromInner}(\text{env}(d))} \text{link} \{(\text{FromEnv}(e'?, \text{to}(d, e')), s')\}} \\ \\ \text{LINK-STEP-WAIT-L-UB} \\ \frac{(L, s, e) \rightsquigarrow \zeta}{(\text{Wait}, s) \xrightarrow{\text{FromInner}(\text{left}(e!, d))} \text{link} \{(\text{Ub}, s)\}} \\ \\ \text{LINK-STEP-WAIT-R-UB} \\ \frac{(R, s, e) \rightsquigarrow \zeta}{(\text{Wait}, s) \xrightarrow{\text{FromInner}(\text{right}(e!, d))} \text{link} \{(\text{Ub}, s)\}} \\ \\ \text{LINK-STEP-WAIT-N-UB} \\ \frac{(E, s, e) \rightsquigarrow \zeta}{(\text{Wait}, s) \xrightarrow{\text{FromInner}(\text{env}(d))} \text{link} \{(\text{Ub}, s)\}} \\ \\ \text{LINK-STEP-TO-ENV} \\ \frac{(\text{ToEnv}(e, \sigma), s) \rightsquigarrow \zeta}{(\text{ToEnv}(e, \sigma), s) \xrightarrow{\text{ToEnv}(e)} \text{link} \{(\sigma, s)\}} \\ \\ \text{LINK-STEP-FROM-ENV} \\ \frac{(\text{FromEnv}(e, \sigma), s) \rightsquigarrow \zeta}{(\text{FromEnv}(e, \sigma), s) \xrightarrow{\text{FromEnv}(e)} \text{link} \{(\sigma, s)\}} \\ \\ \text{LINK-STEP-TO-INNER} \\ \frac{(\text{ToInner}(e), s) \rightsquigarrow \zeta}{(\text{ToInner}(e), s) \xrightarrow{\text{ToInner}(e)} \text{link} \{(\text{Wait}, s)\}} \\ \\ \text{LINK-STEP-UB} \\ (\text{Ub}, s) \xrightarrow{\tau} \text{link} \emptyset \end{array}$$

Fig. 2. Definition of  $\rightarrow_{\text{link}}$ .

Intuitively,  $\text{wrap}(s_2, F_2)$  works as follows: Given an initial state  $s_2$  and a proposition describing resource ownership of the translation  $F_2$ ,  $\text{wrap}$  synchronizes with the environment on an event  $e_2$ . Then it angelically chooses an event  $e_1$  for the inner module, a new state  $s_1$ , ownership of the environment  $F_1$ , and a proof that the ownership of the translation together with the ownership of the environment and the precondition  $(e_1, s_1) \leftarrow (e_2, s_2)$  is satisfiable. Then  $\text{wrap}$  sends  $e_1$  to the inner module  $M$ . Next, it receives an event  $e'_1$  from  $M$  and (demonically) chooses an event  $e'_2$  to emit to the environment, a new state  $s'_2$ , new ownership of the translation  $F'_2$ , and a proof that the ownership of the translation together with the ownership of the environment and the postcondition  $(e'_1, s_1) \rightarrow (e'_2, s'_2)$  is satisfiable. After emitting  $e'_2$ , the process repeats with state  $s'_2$  and  $F'_2$ .

## B MICRO-INSTRUCTIONS OF $\text{Asm}$

Inspired by Sammler et al. [2022], instructions  $\mathbf{c}$  in  $\text{Asm}$  are sequences of *micro instructions* (i.e., simple instructions that, when composed together, form an actual instruction), depicted in Fig. 3. The instruction  $\text{sycall}; \mathbf{c}$  does a  $\text{sycall}$  and then executes  $\mathbf{c}$ . The instruction  $\text{upd}(\mathbf{x}, \mathbf{r}, \mathbf{v}); \mathbf{c}$  updates

<sup>2</sup>The Coq development defines an equivalent module directly using a step relation, but we give the definition here using Spec for readability.

**Instr**  $\ni c \triangleq \text{syscall}; c \mid \text{upd}(\mathbf{x}, \mathbf{r}. \mathbf{v}); c \mid \text{ldr}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{v}. \mathbf{v}'); c \mid \text{str}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{v}. \mathbf{v}'); c \mid \text{jump}$

Fig. 3. Micro-Instructions of **Asm**

$$\begin{array}{c}
\text{ASM-LINK-JUMP} \\
\frac{(d' = L \wedge \mathbf{r}(\text{pc}) \in d_1) \vee (d' = R \wedge \mathbf{r}(\text{pc}) \in d_2) \vee (d' = E \wedge \mathbf{r}(\text{pc}) \notin d_1 \cup d_2) \quad d \neq d'}{(d, \text{None}, \mathbf{Jump}(\mathbf{r}, \mathbf{m})) \rightsquigarrow_{d_1, d_2} (d', \text{None}, \mathbf{Jump}(\mathbf{r}, \mathbf{m}))} \\
\text{ASM-LINK-SYSCALL} \\
\frac{d \neq E}{(d, \text{None}, \mathbf{Syscall}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{m})) \rightsquigarrow_{d_1, d_2} (E, \text{Some}(d), \mathbf{Syscall}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{m}))} \\
\text{ASM-LINK-SYSCALL-RETURN} \\
\frac{d' \neq E}{(E, \text{Some}(d'), \mathbf{SyscallRet}(\mathbf{v}, \mathbf{m})) \rightsquigarrow_{d_1, d_2} (d', \text{None}, \mathbf{SyscallRet}(\mathbf{v}, \mathbf{m}))}
\end{array}$$

Fig. 4. Definition of semantic linking relation  $\rightsquigarrow$  for **Asm**.

the register  $\mathbf{x}$  according to the map  $\mathbf{r} \mapsto \mathbf{v}$  applied to the current register values  $\mathbf{r}$  and then executes  $\mathbf{c}$ . The instruction  $\text{ldr}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{v}. \mathbf{v}'); \mathbf{c}$  takes the value stored in  $\mathbf{x}_2$ , applies the transformation  $\mathbf{v} \mapsto \mathbf{v}'$  to it to obtain an address, loads from the memory at that address, stores the result in  $\mathbf{x}_1$ , and then executes  $\mathbf{c}$ . The instruction  $\text{ldr}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{v}. \mathbf{v}'); \mathbf{c}$  takes the value stored in  $\mathbf{x}_2$ , applies the transformation  $\mathbf{v} \mapsto \mathbf{v}'$  to it to obtain an address, stores in the memory at that address the value in  $\mathbf{x}_1$ , and then executes  $\mathbf{c}$ . The instruction  $\text{jump}$  reads the  $\text{pc}$  register and then jumps to the address stored there.

The reason for the micro instruction representation is that we can represent a large instruction set by chaining few primitives. For example, the instructions used in **print** and **locle** are derived as follows:

$$\begin{array}{l}
\mathbf{ret} \triangleq \text{upd}(\text{pc}, \mathbf{r}. \mathbf{r}(\mathbf{x}30)); \text{jump} \quad \mathbf{syscall} \triangleq \text{syscall}; \text{next} \quad \mathbf{mov} \mathbf{x}, \mathbf{v} \triangleq \text{upd}(\mathbf{x}, \mathbf{r}. \mathbf{v}); \text{next} \\
\mathbf{sle} \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \triangleq \text{upd}(\mathbf{x}_1, \mathbf{r}. \text{if } \mathbf{r}(\mathbf{x}_2) \leq \mathbf{r}(\mathbf{x}_3) \text{ then } 1 \text{ else } 0); \text{next}
\end{array}$$

where we abbreviate  $\text{next} \triangleq \text{upd}(\text{pc}, \mathbf{r}. \mathbf{r}(\text{pc}) + 1); \text{jump}$ .

### C SEMANTIC LINKING FOR **Asm**

The full definition of the semantic linking relation  $\rightsquigarrow$  for **Asm** can be found in Fig. 4. Compared to the excerpt shown in the paper, it contains two additional cases, **ASM-LINK-SYSCALL** and **ASM-LINK-SYSCALL-RETURN**. The rule **ASM-LINK-SYSCALL** makes sure syscalls are passed on to the environment (and never come from the environment). When a syscall is triggered, we store the current turn  $d$  in the private state of the linking operator. This way, we can make sure that when we return from a syscall (**ASM-LINK-SYSCALL-RETURN**), the execution continues with the module that triggered the syscall.

### D **Rec**

The language **Rec** is a simple, high-level language with arithmetic operations, let bindings, memory operations, conditionals, and (potentially recursive) function calls (depicted in Fig. 5). The libraries  $R$  of **Rec** are lists of function declarations. Each function declaration contains the name of the function  $f$ , the argument names  $\bar{x}$ , local variables  $\bar{y}$  which are allocated in the memory, and a

$$\begin{aligned}
\text{Library } \ni \mathbf{R} &\triangleq (\text{fn } f(\bar{x}) \triangleq \overline{\text{local } y[n]; e}, \mathbf{R} \mid \emptyset \\
\text{Expr } \ni \mathbf{e} &\triangleq \mathbf{v} \mid \mathbf{x} \mid \mathbf{e}_1 \oplus \mathbf{e}_2 \mid \text{let } \mathbf{x} := \mathbf{e}_1 \text{ in } \mathbf{e}_2 \mid \text{if } \mathbf{e}_1 \text{ then } \mathbf{e}_2 \text{ else } \mathbf{e}_3 \mid \mathbf{e}_1(\overline{\mathbf{e}_2}) \mid !\mathbf{e} \mid \mathbf{e}_1 \leftarrow \mathbf{e}_2 \\
\text{BinOp } \ni \oplus &\triangleq + \mid < \mid == \mid \leq \\
\text{Runtime Expr } \ni \mathbf{E} &\triangleq \dots \mid \text{alloc\_frame } (\overline{\mathbf{x}}, n) \mathbf{E} \mid \text{free\_frame } (\overline{\ell}, n) \mathbf{E} \mid \text{Ret}(b, \mathbf{E}) \mid \text{Wait}(b)
\end{aligned}$$

Fig. 5. Grammar of **Rec**.

function body  $e$ . The set of function names  $|\mathbf{R}|$  of a library  $\mathbf{R}$  is defined as the names of the functions in the list  $\mathbf{R}$ .

**Module semantics.** The semantics of a **Rec** library  $\mathbf{R}$  is the module  $\llbracket \mathbf{R} \rrbracket_r$ . The states of the module are of the form  $\sigma = (\mathbf{E}, \mathbf{m}, \mathbf{R})$  where  $\mathbf{E}$  is the current *runtime expression* (explained below). We write  $(\rightarrow_r)$  for the transition system (shown in Fig. 6) and the initial state is  $(\text{Wait}(\text{false}), \emptyset, \mathbf{R})$ .

To define the transition relation  $\rightarrow_r$ , we extend the static expressions  $e$  to runtime expressions  $\mathbf{E}$ , which have operations for allocating and deallocating stack frames as well as two distinguished expressions  $\text{Ret}(b, \mathbf{E})$  and  $\text{Wait}(b)$ . These expressions are used to control when the module emits call and return events: Initially, the module is waiting and willing to accept any incoming call to the functions of the library (see **REC-START**). Once it starts, the function call is wrapped in the  $\text{Ret}(b, \cdot)$  expression to ensure an event is emitted after the function finishes executing (see **REC-RET-RETURN**). A call to functions of the library (see **REC-CALL-INTERNAL**), will trigger the allocation of the local variables and, subsequently, the execution of the function body. A call to an external function (see **REC-CALL-EXTERNAL**) will emit a  $\text{Call}!(f, \bar{v}, \mathbf{m})$  and proceed to the waiting state. The flag for the waiting becomes true, because the module is now willing to accept a return to the function call that was just issued (see **REC-RET-INCOMING**). The language **Rec** is an evaluation-context based language, meaning reductions can happen inside of an arbitrary evaluation context (see **REC-EVAL-CTX**). The definition of the evaluation contexts  $\mathbf{K}$  can be found in the Coq development [Sammler et al. 2023].

**Linking.** Syntactically, linking of two **Rec** libraries (*i.e.*,  $\mathbf{R}_1 \cup_r \mathbf{R}_2$ ) denotes merging the function definitions in  $\mathbf{R}_1$  and  $\mathbf{R}_2$ . In case of overlapping function names, the function declaration of the left library is chosen. (This choice is arbitrary.) If we semantically link two **Rec** modules (*i.e.*,  $M_1 \overset{d_1}{\oplus}_r \overset{d_2}{M_2}$ ), then we have to synchronize based on the function call and return events. To define the linking  $M_1 \overset{d_1}{\oplus}_r \overset{d_2}{M_2}$ , we use the combinator  $M_1 \oplus_X M_2$ . In the case of **Rec**, we pick the relation  $\mathbf{R}$  depicted in Fig. 7. The most interesting difference to **Asm** is that linking in **Rec** has to build up and then wind down a call-stack, which is maintained as the internal state of  $(\rightsquigarrow)$ .

## $\mathbf{E} \llbracket \cdot \rrbracket_{r \Leftarrow a}$ WRAPPER

Before we can give the definition of the wrapper  $\llbracket \cdot \rrbracket_{r \Leftarrow a}$ , we first need to describe its full form:  $\llbracket M \rrbracket_{r \Leftarrow a}^{a, d, d, m}$ . In particular, the wrapper is parametrized by a mapping  $a_-$  from **Rec** function names to **Asm** addresses, by the instruction address of the **Asm** code  $d$ , by the function names of the **Rec** code  $d$ , and by a (fragment of) the initial memory  $m$ , which can be used for global variables.

To define the wrapper, we pick a suitable flavor of separation logic. Instead of directly presenting the technical details of the resource algebra that we choose for  $\mathcal{R}_{r \Leftarrow a}$ , we instead describe the connectives of the resulting separation logic:

- $\mathbf{p} \leftrightarrow \mathbf{v}$  states that the **Rec** block id  $\mathbf{p}$  is mapped to **Asm** address  $\mathbf{v}$ . We lift this relation to locations by  $\ell \leftrightarrow \mathbf{v}_2 \triangleq \exists \mathbf{v}_1. \ell.\text{blockid} \leftrightarrow \mathbf{v}_1 * \mathbf{v}_2 = \mathbf{v}_1 + \ell.\text{offset}$  and to values (*i.e.*,  $\mathbf{v} \leftrightarrow \mathbf{v}$ ) by

$$\begin{array}{c}
\text{REC-BINOP} \\
(v_1 \oplus v_2, m, R) \xrightarrow{\tau}_r \{(v, m, R) \mid \text{eval}_{\oplus}(v_1, v_2, v)\} \\
\\
\text{REC-LOAD} \\
(!v_1, m, R) \xrightarrow{\tau}_r \{(v_2, m, R) \mid \exists \ell. v_1 = \ell \wedge m(\ell) = v_2\} \\
\\
\text{REC-STORE} \\
(v_1 \leftarrow v_2, m, R) \xrightarrow{\tau}_r \{(v_2, m[\ell \mapsto v_2], R) \mid \exists \ell. v_1 = \ell \wedge \text{heap\_alive}(m, \ell)\} \\
\\
\text{REC-IF} \\
(\text{if } v \text{ then } e_1 \text{ else } e_2, m, R) \xrightarrow{\tau}_r \{(e, m, R) \mid \exists b. v = b \wedge \text{if } b \text{ then } e = e_1 \text{ else } e = e_2\} \\
\\
\begin{array}{cc}
\text{REC-LET} & \text{REC-VAR} \\
(\text{let } x := v \text{ in } e, m, R) \xrightarrow{\tau}_r \{(e[v/x], m, R)\} & (x, m, R) \xrightarrow{\tau}_r \emptyset
\end{array} \\
\\
\text{REC-ALLOC} \\
\frac{\text{heap\_alloc\_list}(\bar{n}, \bar{\ell}, m_1, m_2)}{(\text{alloc } \overline{(y, n)} e, m_1, R) \xrightarrow{\tau}_r \{(\text{free\_frame } \overline{(\ell, n)} (e[\bar{\ell}/\bar{y}]), m_2, R) \mid \forall m \in \bar{n}. m > 0\}} \\
\\
\text{REC-FREE} \\
(\text{free\_frame } \overline{(\ell, n)} v, m_1, R) \xrightarrow{\tau}_r \{(v, m_2, R) \mid \text{heap\_free\_list}(\overline{(\ell, n)}, m_1, m_2)\} \\
\\
\text{REC-START} \\
\frac{f \in R}{(\text{Wait}(b), m, R) \xrightarrow{\text{Call}?(f, \bar{v}, m')} \{(\text{Ret}(b, f(\bar{v})), m', R)\}} \\
\\
\text{REC-CALL-INTERNAL} \\
\frac{(\text{fn } f(\bar{x}) \triangleq \text{local } y[n]; e) \in R}{(f(\bar{v}), m, R) \xrightarrow{\tau}_r \{(\text{alloc } \overline{(y, n)} (e[\bar{v}/\bar{x}]), m, R) \mid |\bar{x}| = |\bar{v}|\}} \\
\\
\text{REC-CALL-EXTERNAL} \\
\frac{f \notin R}{(f(\bar{v}), m, R) \xrightarrow{\text{Call}!(f, \bar{v}, m)} \{(\text{Wait}(\text{true}), m, R)\}} \quad \text{REC-RET-INCOMING} \\
(\text{Wait}(\text{true}), m, R) \xrightarrow{\text{Return}?(v, m')} \{(v, m', R)\} \\
\\
\text{REC-RET-RETURN} \\
(\text{Ret}(b, v), m, R) \xrightarrow{\text{Return}!(v, m)} \{(\text{Wait}(b), m, R)\} \\
\\
\text{REC-EVAL-CTX} \\
\frac{(E, m, R) \xrightarrow{\alpha}_r \Sigma}{(K[E], m, R) \xrightarrow{\alpha}_r \{(K[E'], m', R') \mid (E', m', R') \in \Sigma\}}
\end{array}$$

Fig. 6. Operational semantics of `Rec`.

relating `Rec` integers with the same integer in `Asm` and Boolean values with 0 and 1. The definition of  $v \leftrightarrow v$  corresponds to  $v \sim_w v$  in the main paper.

$$\begin{array}{c}
\text{REC-LINK-CALL} \\
\dfrac{(d' = L \wedge f \in d_1) \vee (d' = R \wedge f \in d_2) \vee (d' = E \wedge f \notin d_1 \cup d_2) \quad d \neq d'}{(d, \bar{d}_s, \text{Call}(f, \bar{v}, m)) \rightsquigarrow_{d_1, d_2} (d', d :: \bar{d}_s, \text{Call}(f, \bar{v}, m))} \\
\\
\text{REC-LINK-RET} \\
\dfrac{d \neq d'}{(d, d' :: \bar{d}_s, \text{Return}(v, m)) \rightsquigarrow_{d_1, d_2} (d', \bar{d}_s, \text{Return}(v, m))}
\end{array}$$

Fig. 7. Definition of semantic linking relation  $\rightsquigarrow_{d_1, d_2}$  for *Rec*.

$$\begin{array}{l}
(e_1, s_1) \rightarrow (e_2, s_2) \triangleq \exists r \, m \, \bar{v}. e_2 = \text{Jump!}(r, m) * \text{inv}(r(\text{sp}), m, \text{mem}(e_1)) * \\
(\exists f \, \bar{v} \, m. e_1 = \text{Call!}(f, \bar{v}, m) * f \notin d * r(x30) \in d * a_f = r(\text{pc}) * \\
s_2 = r :: s_1 * \quad * \quad v \leftrightarrow v \\
v, v \in \bar{v}, \text{take}(|\bar{v}|, r(x0...x8)) \\
\vee \exists v \, m \, r'. e_1 = \text{Return!}(v, m) * r' :: s_2 = s_1 * r(\text{pc}) = r'(x30) * \\
r(x19 \dots x29, \text{sp}) = r'(x19 \dots x29, \text{sp}) * v \leftrightarrow r(x0)) \\
\\
(e_1, s_1) \leftarrow (e_2, s_2) \triangleq \exists r \, m \, \bar{v}. e_2 = \text{Jump?}(r, m) * \text{inv}(r(\text{sp}), m, \text{mem}(e_1)) * \\
(\exists f \, \bar{v} \, m. e_1 = \text{Call?}(f, \bar{v}, m) * f \in d * r(x30) \notin d * a_f = r(\text{pc}) * \\
s_1 = r :: s_2 * \quad * \quad v \leftrightarrow v \\
v, v \in \bar{v}, \text{take}(|\bar{v}|, r(x0...x8)) \\
\vee \exists v \, m \, r'. e_1 = \text{Return?}(v, m) * r' :: s_1 = s_2 * r(\text{pc}) = r'(x30) * \\
r(x19 \dots x29, \text{sp}) = r'(x19 \dots x29, \text{sp}) * v \leftrightarrow r(x0))
\end{array}$$

Fig. 8. Definition of  $(\leftarrow)$  and  $(\rightarrow)$  for  $[\cdot]_{r \Rightarrow a}$ .

- $v_1 \mapsto_a v_2$  asserts ownership of the address  $v_1$  in *Asm* memory  $m$  and asserts that it contains the value  $v_2$ . The  $v_1 \mapsto_a v_2$  connective is useful for asserting private ownership of *Asm* memory in assembly libraries (e.g., it is used internally by the coroutine library to manage its global state).
- $p \mapsto_r V$  where  $V$  is a map from offsets to values asserts that the block with id  $p$  contains exactly  $V$ . The  $p \mapsto_r V$  connective is useful for asserting ownership of locations in the *Rec* memory, e.g., for locations that are not mapped to the *Asm* memory.
- $\text{inv}(v, m, m)$  asserts that  $m$  and  $m$  are in an invariant such that all the aforementioned assertions (i.e.,  $p \leftrightarrow v, v_1 \mapsto_a v_2$ , and  $p \mapsto_r V$ ) have the meaning described above and  $v$  points to a valid stack.

This separation logic is used to define the relations  $(\leftarrow)$  and  $(\rightarrow)$  (depicted in Fig. 8) that are used in the definition of  $[\cdot]_{r \Rightarrow a}$ . Note that these definitions build on the definition of the Kripke wrapper in Appendix A as they maintain the state  $s$  for tracking the call stack in addition to the

$$\begin{array}{c}
\text{CORO-LINK-YIELD} \\
\frac{(d = L \wedge d' = R) \vee (d = R \wedge d' = L)}{(d, (d, \text{None}), \text{Call}(\text{yield}, [v], m)) \rightsquigarrow_{\text{coro}}^{d_1, d_2} (d', (d', \text{None}), \text{Return}(v, m))} \\
\\
\text{CORO-LINK-YIELD-UB} \\
\frac{d = L \vee d = R \quad |\bar{v}| \neq 1}{(d, (d, \text{None}), \text{Call}(\text{yield}, \bar{v}, m)) \rightsquigarrow_{\text{coro}}^{d_1, d_2} \downarrow} \\
\\
\text{CORO-LINK-L-YIELD-INIT} \\
(L, (L, \text{Some}(f)), \text{Call}(\text{yield}, [v], m)) \rightsquigarrow_{\text{coro}}^{d_1, d_2} (R, (R, \text{None}), \text{Call}(f, [v], m)) \\
\\
\text{CORO-LINK-L-YIELD-INIT-UB} \\
\frac{|\bar{v}| \neq 1}{(L, (L, \text{Some}(f)), \text{Call}(\text{yield}, \bar{v}, m)) \rightsquigarrow_{\text{coro}}^{d_1, d_2} \downarrow} \\
\\
\text{CORO-LINK-INIT} \quad \frac{f \in |M_1|}{(E, (E, f^0), \text{Call}(f, \bar{v}, m)) \rightsquigarrow_{\text{coro}} (L, \text{Call}(f, \bar{v}, m), (L, f^0))} \quad \text{CORO-LINK-INIT-UB} \quad \frac{f \notin |M_1|}{(E, (E, f^0), \text{Call}(f, \bar{v}, m)) \rightsquigarrow_{\text{coro}} \downarrow} \\
\\
\text{CORO-LINK-L-RETURN} \quad (L, (L, f^0), \text{Return}(v, m)) \rightsquigarrow_{\text{coro}} (E, (E, f^0), \text{Return}(v, m)) \quad \text{CORO-LINK-R-RETURN} \quad (R, (R, \text{Return}(v, m)) \rightsquigarrow_{\text{coro}} \downarrow \\
\\
\text{CORO-LINK-CALL} \\
\frac{f \neq \text{yield} \quad (d = L \wedge f \notin |M_2|) \vee (d = R \wedge f \notin |M_1|)}{(L, (d, f^0), \text{Call}(f, \bar{v}, m)) \rightsquigarrow_{\text{coro}} (E, (d, f^0), \text{Call}(f, \bar{v}, m))} \\
\\
\text{CORO-LINK-CALL-UB} \\
\frac{f \neq \text{yield} \quad (d = L \wedge f \in |M_2|) \vee (d = R \wedge f \in |M_1|)}{(L, (d, f^0), \text{Call}(f, \bar{v}, m)) \rightsquigarrow_{\text{coro}} \downarrow} \\
\\
\text{CORO-LINK-E-RETURN} \\
\frac{(s = L \wedge d = L) \vee (s = R \wedge d = R) \quad e = \text{Return}(\_, \_)}{(E, (d, f^0), e) \rightsquigarrow_{\text{coro}} (d, (d, f^0), e)} \\
\\
\text{CORO-LINK-E-CALL-UB} \\
\frac{(s = L \wedge d = L) \vee (s = R \wedge d = R) \quad e = \text{Call}(\_, \_, \_)}{(E, (d, f^0), e) \rightsquigarrow_{\text{coro}} \downarrow}
\end{array}$$

Fig. 9. Definition of linking relation  $\rightsquigarrow_{\text{coro}}^{d_1, d_2}$ .

separation logic predicates. We define:

$$\llbracket M \rrbracket_{r \rightleftharpoons a}^{a, \_, d, m} \triangleq \llbracket M \rrbracket_X \quad \text{where} \quad X \triangleq (\text{List}(\mathbf{Registers}), \mathcal{R}_{r \rightleftharpoons a}, \leftarrow, \rightarrow, [], \ast, \mathbf{v}_1 \mapsto_a \mathbf{v}_2)$$

$\mathbf{v}_1 \mapsto_a \mathbf{v}_2 \in \mathbf{M}$

## F COROUTINE LINKING

Formally,  $M_1 \oplus_{\text{coro}} M_2$  is defined using the generic linking operator  $M_1 \oplus_X M_2$ . Concretely, we define  $M_1 \overset{d_1}{\oplus}_{\text{coro}} \overset{d_2, f}{\oplus} M_2 \triangleq M_1 \oplus_{X_{\text{coro}}} M_2$  where

$$X_{\text{coro}} \triangleq ((D \times \text{option}(\text{FnName})), \rightsquigarrow_{\text{coro}}^{d_1, d_2}, (E, \text{Some}(f)))$$

Note that this linking operator is parametrized by a function name  $f$  of the initial function on the right side of the linking (`stream` in the example). The effect of linking is described by  $\rightsquigarrow_{\text{coro}}$  shown in Fig. 9. There are many transitions, but most of them are straightforward. The rule `CORO-LINK-YIELD` encodes the core idea of  $\oplus_{\text{coro}}$ : If either the left side or the right side performs a call to `yield`, control switches to the other side, and the event is transformed to a `Return?(v, m)` event. There is one special case to consider: When  $M_1$  calls `yield` the first time, there is no `yield` in  $M_2$  from which to return. Instead this first call to `yield` becomes the invocation of a designated start function  $f$  in  $M_2$  (`stream` in the example), as stated by `CORO-LINK-L-YIELD-INIT`. `CORO-LINK-INIT` handles the initial call from the environment to  $M_1$ . If the environment tries to call a function not in  $M_1$ , the behavior is undefined (`CORO-LINK-INIT-UB`). `CORO-LINK-L-RETURN` handles the return from  $M_1$  to the environment.  $M_2$  should never return and thus `CORO-LINK-R-RETURN` states that doing so would lead to undefined behavior. Finally, `CORO-LINK-CALL` and `CORO-LINK-E-RETURN` allow both  $M_1$  and  $M_2$  to call external functions (like `print`). However,  $M_1$  and  $M_2$  cannot directly call a function in the other module (without going through `yield`) (`CORO-LINK-CALL-UB`) and the environment may not call them back recursively (`CORO-LINK-CALL-UB`).

## REFERENCES

- Ralf Jung, Robbert Krebbers, Jacques-Henri Jourdan, Ales Bizjak, Lars Birkedal, and Derek Dreyer. 2018. Iris from the ground up: A modular foundation for higher-order concurrent separation logic. *J. Funct. Program.* 28 (2018), e20. <https://doi.org/10.1017/S0956796818000151>
- Ralf Jung, David Swasey, Filip Sieczkowski, Kasper Svendsen, Aaron Turon, Lars Birkedal, and Derek Dreyer. 2015. Iris: Monoids and Invariants as an Orthogonal Basis for Concurrent Reasoning. In *POPL*. ACM, 637–650. <https://doi.org/10.1145/2676726.2676980>
- Michael Sammler, Angus Hammond, Rodolphe Lepigre, Brian Campbell, Jean Pichon-Pharabod, Derek Dreyer, Deepak Garg, and Peter Sewell. 2022. Islaris: verification of machine code against authoritative ISA semantics. In *PLDI*. ACM, 825–840. <https://doi.org/10.1145/3519939.3523434>
- Michael Sammler, Simon Spies, Youngju Song, Emanuele D’Osualdo, Robbert Krebbers, Deepak Garg, and Derek Dreyer. 2023. DimSum: A Decentralized Approach to Multi-language Semantics and Verification (Coq development). <https://doi.org/10.5281/zenodo.7306312> Project webpage: <https://plv.mpi-sws.org/dimsum/>.